

# LAGRANGE MULTIPLIER SELECTION FOR RATE-DISTORTION OPTIMIZATION IN SVC

Xiang Li<sup>1,2</sup>, Peter Amon<sup>2</sup>, Andreas Hutter<sup>2</sup>, and André Kaup<sup>1</sup>

<sup>1</sup>Chair of Multimedia Communications and Signal Processing,  
University of Erlangen-Nuremberg, Erlangen, Germany

<sup>2</sup>Siemens Corporate Technology, Information & Communications, Munich, Germany

## ABSTRACT

The Lagrangian multiplier based rate-distortion optimization (RDO) has been widely employed in single layer video coding. During the development of scalable video coding (SVC) extension of H.264/AVC, it was directly applied in a multi-layer scenario. However, such an application is not very efficient since the correlation between layers is not considered in the Lagrange multiplier selection. To improve the overall performance, in this paper a new selection algorithm is presented for RDO in SVC. Simulations show that the proposed method outperforms the recent SVC reference software. With a tiny computational cost, average gains of 0.22 dB and 0.35 dB were achieved in the tests of four-layer quality scalability and three-layer spatial scalability, respectively.

**Index Terms**— SVC, Rate-Distortion Optimization, Lagrange Multiplier Selection

## 1. INTRODUCTION

Rate-Distortion Optimization (RDO) techniques have been widely used in today's single layer video coding. Its target is to minimize the distortion  $D$  for a given rate  $R_c$  by appropriate selections of coding parameters, namely

$$\begin{aligned} & \min\{D\} \\ & \text{subject to } R \leq R_c \end{aligned} \quad (1)$$

However, (1) is a constrained problem which is hard to solve. Therefore, (1) is converted to (2) by Lagrange multiplier method,

$$\begin{aligned} & \min\{J\} \\ & \text{where } J = D + \lambda \cdot R \end{aligned} \quad (2)$$

where  $J$  denotes the Lagrangian cost function and  $\lambda$  is the so-called Lagrange multiplier. Consequently, how to determine  $\lambda$  becomes a key problem in Lagrangian RDO.

Supposing  $R$  and  $D$  to be differentiable everywhere, the minimum cost  $J$  is given by setting its derivative to zero, i.e.,

$$\frac{dJ}{dR} = \frac{dD}{dR} + \lambda = 0 \quad (3)$$

leading to

$$\lambda = -\frac{dD}{dR} \quad (4)$$

Assuming a sufficiently high rate environment, [1] proposed the rate model  $R_S$  and distortion model  $D_S$  for single layer video coding, namely

$$\begin{aligned} R_S &= a \log_2\left(\frac{b}{D_S}\right) \\ D_S &= \frac{Q^2}{12} \end{aligned} \quad (5)$$

where  $a$  and  $b$  are two constants,  $Q$  is the quantization step.

When putting (5) into (4),  $\lambda$  for a single layer is determined by

$$\lambda = -\frac{dD}{dR} = c \cdot Q^2 \quad (6)$$

where  $c$  is a constant which is experimentally suggested to be 0.85 [1], though others proposed 0.68 [2].

In practice, this  $\lambda$  selection method is simple and efficient. It was adopted into H.264/AVC reference software for single layer video coding [3]. Moreover, during the development of scalable video coding (SVC) extension of H.264/AVC, it was also applied in the joint scalable video model (JSVM) [4]. However, such a direct application of (6) in a SVC scenario is not very efficient since the correlation between layers is not considered in (6).

Recently, an empirical multi-layer  $\lambda$  selection method was proposed for quality scalable video coding [5]. However, its computational complexity is high due to the multi-pass regression process. In addition, [6] proposed a *multi-pass* RDO method for SVC where the two neighboring layers were joint optimized while keeping the  $\lambda$  for each layer unchanged. Although this algorithm is quite efficient, its computational complexity is quite high. To keep a low computational cost while achieve a good performance in both quality scalability and spatial scalability, a new  $\lambda$  selection algorithm is presented in this paper. Compared with the recent SVC reference software JSVM 9.13.1, the proposed algorithm shows a higher coding efficiency at a tiny computational cost. For a four-layer quality scalability environment, an average gain of 0.22 dB was obtained for eight sequences. While for a three-layer spatial scalability case, 0.35 dB gain on average was achieved for four sequences.

The rest of this paper is organized as follows. First in Section 2, the proposed method is depicted in detail. Then

in Section 3, simulation results and related discussions are presented. Finally the whole paper is concluded in Section 4.

## 2. LAGRANGE MULTIPLIER SELECTION IN SVC

In this section, the proposed  $\lambda$  selection method is first discussed. Then the implementation details are provided.

### 2.1. Multi-Layer Lagrange Multiplier Selection

In a SVC scenario, all the layers should be jointly optimized in order to achieve the best performance. Without loss of generality, a two-layer scenario is studied in this paper for simplicity. In such a case, the task of RDO is extended as

$$\min\{J\} = \min\{(1-w) \cdot J_0 + w \cdot J_1\} \quad (7)$$

where  $J_0$  and  $J_1$  represent R-D cost for the base layer and enhancement layer, and  $w \in [0, 1]$  is a layer weighting factor. Intuitively, when  $w = 0$  or  $w = 1$ , (7) reduces to the single layer RDO for the base layer or the enhancement layer, respectively.

More concretely,  $J$  in (7) is written to

$$J = (1-w) \cdot (D_0 + \lambda_0 \cdot R_0) + w \cdot (D_1 + \lambda_1 \cdot (R_0 + R_1)) \quad (8)$$

where  $R_0$  is included in the second term (the calculation of  $J_1$ ) since in SVC the accumulated bitrate instead of self bitrate for a layer is used to evaluate the coding efficiency.

Similar to the single layer RDO, the minimum  $J$  in (8) can be obtained by setting its derivative to zero. To calculate this, R-D models are needed. Besides those in (5), there are other models proposed in the literature, such as the Laplace distribution based models in our previous work [7]. Although our Laplacian models show a better accuracy, those in (5) are employed in this paper to ease the comparison with the reference software and demonstrate the gain is directly from the proposed algorithm.

Considering (5) actually describes the rate and distortion at pixel level and no frame size information is counted, a resolution factor has to be introduced when applying these models in a spatial scalability scenario. Therefore, we propose to calculate the joint cost  $J$  as

$$J = (1-w) \cdot (D(Q+\Delta) + \lambda_0 \cdot R(Q+\Delta)) + w \cdot (D(Q) \cdot \beta + \lambda_1 \cdot (R(Q+\Delta) + R(Q) \cdot \beta)) \quad (9)$$

where  $D(\cdot)$  and  $R(\cdot)$  represent the models in (5),  $(Q+\Delta)$  and  $Q$  are quantization steps for the base layer and enhancement layer, respectively, and  $\beta$  denotes the resolution ratio between the two layers. For example,  $\beta = 1$  describes a same-resolution case, while  $\beta = 4$  may indicate a QCIF-CIF environment. As well known, for the same content at different resolutions, the bitrate ratio may be approximated by the resolution ratio although the two ratios are not precisely the same. In this paper, such an approximation is directly employed for

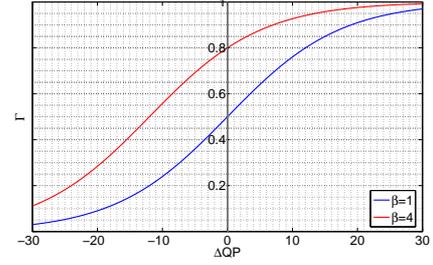


Fig. 1. Relationship among  $\Gamma$ ,  $\Delta QP$  and  $\beta$

simplicity. For further improvement, more accurate models can be applied to compensate this problem.

Putting (5) into (9) and then setting the derivative of  $J$  to zero,  $\lambda_1$  can be solved as

$$\lambda_1 = \frac{Q((1-w)(\ln(2)(Q+\Delta)^2 - 12a\lambda_0) + \ln(2)\beta w Q(Q+\Delta))}{12aw(\beta(Q+\Delta) + Q)} \quad (10)$$

If  $\lambda_0$  is determined by the single layer  $\lambda$  selection method (6), i.e., plugging  $\lambda_0 = c \cdot (Q+\Delta)^2$  into (10),  $\lambda_1$  is derived as

$$\lambda_1 = \frac{\beta \cdot (Q+\Delta)}{\beta \cdot (Q+\Delta) + Q} \cdot (c \cdot Q^2) \quad (11)$$

Considering the last term in (11) is actually the  $\lambda$  by the single layer selection method (6),  $\lambda_1$  is written to

$$\lambda_1 = \Gamma \cdot \lambda(Q) \quad (12)$$

where  $\Gamma$  is defined in (13) and  $\lambda(Q)$  denotes the single layer  $\lambda$  function by (6).

$$\Gamma = \frac{\beta \cdot (Q+\Delta)}{\beta \cdot (Q+\Delta) + Q} \quad (13)$$

According to the definition in H.264/AVC [3, 4, 8], the quantization steps for the base and enhancement layers are derived as

$$\begin{aligned} Q+\Delta &= 2.5 \cdot 2^{(QP_0-12)/6} \\ Q &= 2.5 \cdot 2^{(QP_1-12)/6} \end{aligned} \quad (14)$$

where  $QP_0$  and  $QP_1$  are quantization parameters for the base and enhancement layers, respectively.

Taking (14) into (13),  $\Gamma$  is simplified to

$$\Gamma = \frac{\beta \cdot 2^{\Delta QP/6}}{\beta \cdot 2^{\Delta QP/6} + 1} \quad (15)$$

where  $\Delta QP = QP_0 - QP_1$  which describes the difference between the quantization parameters for the two layers.

Fig. 1 shows two  $\Delta QP$ - $\Gamma$  curves with different  $\beta$ . For a given  $\beta$ ,  $\Gamma$  is monotonously increasing with  $\Delta QP$ . When  $\Delta QP$  is very big,  $\Gamma$  approaches 1 and  $\lambda_1$  approaches  $\lambda$  of single layer. This is reasonable since a very big  $\Delta QP$  will result in a very low correlation between the base and enhancement layers so that the optimal solution is to optimize the two layers independently. Contrariwise, when  $\Delta QP$  is too small,  $\Gamma$

approaches 0, which will lead to a very small  $\lambda_1$ . In practice, it is a very rare condition since the PSNR quality of the enhancement layer will be much worse than the base layer due to the much larger quantization step. In such a case, the rate of the enhancement layer is negligible when compared with that of the base layer, and the quality is much more important. Thus a very small  $\lambda_1$  is necessary. On the other hand, in the environment of spatial scalability, bigger  $\beta$  will lead to a bigger  $\Gamma$ . This is also easy to understand. When the resolution of the enhancement layer is much higher than that of the base layer, more weighting is put on the enhancement layer. Consequently, the joint optimization is shifted towards the independent RDO for the enhancement layer.

## 2.2. Implementation Details

Basically, (12) formulates the proposed multi-layer Lagrange multiplier selection method ML- $\lambda$  where a new factor  $\Gamma$  is introduced. (13) shows that the computational complexity for  $\Gamma$  is quite low. Considering all the calculations in (12) occur at frame level, the total computational cost by the proposed algorithm is rather marginal.

Although the proposed algorithm is derived in a two-layer scenario, it can be easily extended to a multi-layer environment by counting the costs of other layers in (7). However in practice, there is an even simpler way. Since simulations indicate that the correlation between two non-neighboring layers is comparatively low,  $\lambda_n$  normally shows little impact on the layer  $n + 2$  and higher layers. Therefore a two-layer sliding window process is employed. That is for layer  $n$ ,  $\Gamma_n$  is first derived based on  $\Delta QP$  and  $\beta$  between its base layer and itself. Then  $\lambda_n$  is calculated according to (12). Since there is no base layer for layer 0, in the current implementation  $\lambda_0$  is unchanged. Consequently, the performance of the layer 0 is exactly the same as that of the JSVM software.

As mentioned in Section 1, there were two values proposed for the constant  $c$  in (6). To cover the both cases,  $\Gamma'$  is defined as in (16), i.e.,  $\Gamma$  and  $\Gamma'$  correspond to  $c = 0.85$  and  $c = 0.68$ , respectively.

$$\Gamma' = \frac{0.68}{0.85} \cdot \Gamma = 0.8 \cdot \Gamma \quad (16)$$

In the following simulations, the performance of ML- $\lambda$  with both  $\Gamma$  and  $\Gamma'$  will be evaluated. For convenience, they will be referenced as ML- $\lambda(\Gamma)$  and ML- $\lambda(\Gamma')$ , respectively.

## 3. SIMULATIONS AND DISCUSSIONS

The proposed algorithm was verified by the recent SVC reference software JSVM 9.13.1 [4] in the environments of quality scalability and spatial scalability. Basically, IPPP coding structure (only one I frame at the very beginning), CABAC, fast search algorithm for motion estimation were enabled while temporal scalability, medium granularity scalability, 8x8 transform, and low complexity MB mode were disabled. To evaluate the overall coding efficiency, the gain

**Table 1.** Gains over JSVM 9.13.1 (dB)

sequences	Quality Scalability		Spatial Scalability					
	ML- $\lambda$ ( $\Gamma$ )	ML- $\lambda$ ( $\Gamma'$ )	ML- $\lambda$ ( $\Gamma$ )			ML- $\lambda$ ( $\Gamma'$ )		
	$\Delta P$	$\Delta P$	$\Delta P_1$	$\Delta P_2$	$\Delta P_S$	$\Delta P_1$	$\Delta P_2$	$\Delta P_S$
<i>bus</i>	0.17	0.19	-	-	-	-	-	-
<i>football</i>	0.20	0.23	-	-	-	-	-	-
<i>foreman</i>	0.18	0.21	-	-	-	-	-	-
<i>mobile</i>	0.20	0.24	-	-	-	-	-	-
<i>city</i>	0.23	0.24	0.17	0.13	0.30	0.24	0.16	0.40
<i>crew</i>	0.17	0.19	0.09	0.10	0.19	0.15	0.14	0.28
<i>harbour</i>	0.17	0.18	0.12	0.10	0.22	0.16	0.13	0.29
<i>soccer</i>	0.24	0.28	0.15	0.14	0.29	0.22	0.19	0.41
average	0.19	0.22	0.13	0.12	0.25	0.19	0.16	0.35

$\Delta P$  in PSNR-Y over the JSVM software is calculated according to [9].

### 3.1. Quality Scalability

In this sub-section, the performance of the proposed algorithm is evaluated in the environment of quality scalability. Eight sequences defined in [10] are coded. To cover a practical quality range, four layers are employed with fixed quantization parameters, i.e.,  $QP = (36, 32, 28, 24)$ . Clearly in this test  $\Delta QP = 4$  and  $\beta = 1$ . According to (15) and (16),  $\Gamma = 0.61$  while  $\Gamma' = 0.49$ .

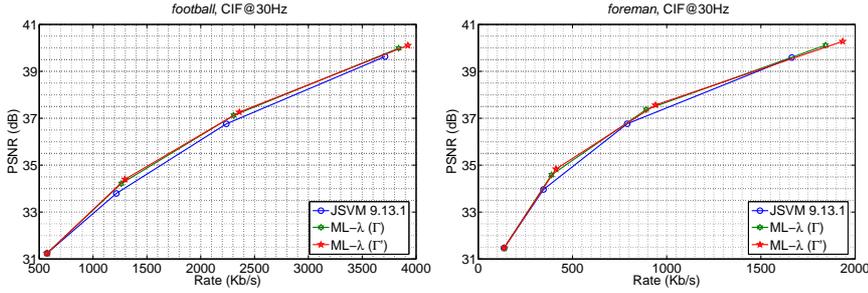
Table 1 (left part) presents the simulation results for quality scalability and Fig. 2 show two related R-D curves. On average, 0.19 dB and 0.22 dB gains over JSVM software are achieved by ML- $\lambda$  ( $\Gamma$ ) and ML- $\lambda$  ( $\Gamma'$ ), respectively. Moreover, it can be observed from the table and figure that the gains are not only evenly distributed among the eight sequences, but also evenly distributed within the bitrate range for each sequence, which indicates the proposed algorithm is able to well adapt to different scenarios and different bitrates.

Moreover, to check whether the theoretical value  $\Gamma = 0.61$  ( $\Gamma' = 0.49$ ) is optimal, simulations on different  $\Gamma$  values were conducted. Fig. 3 investigates the relationship between  $\Gamma$  value and the average  $\Delta P$  for the same eight sequences. As shown in the figure, when  $\Gamma' = 0.49$ , the best performance is achieved, which well matches the theory. In addition, the performance of ML- $\lambda$  ( $\Gamma$ ) is a little worse than ML- $\lambda$  ( $\Gamma'$ ), which indicates in quality scalable environment 0.68 is a better value than 0.85 for  $c$  in (6).

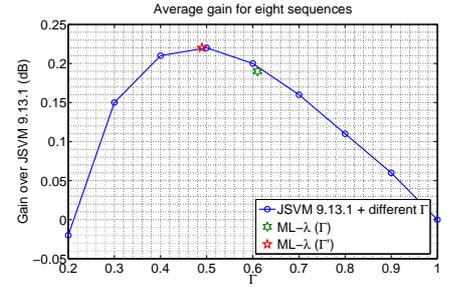
### 3.2. Spatial Scalability

In this sub-section, the performance of the proposed algorithm is evaluated in the environment of spatial scalability. Four sequences defined in [10] are tested in a similar way to [11], i.e., each of them is coded into three spatial layers where  $QP_0 = (34, 30, 26, 22)$ ,  $QP_1 = (36, 32, 28, 24)$ , and  $QP_2 = (38, 34, 30, 26)$ . In such an environment,  $\Delta QP = -2$  and  $\beta = 4$ . Consequently,  $\Gamma = 0.76$  while  $\Gamma' = 0.61$ .

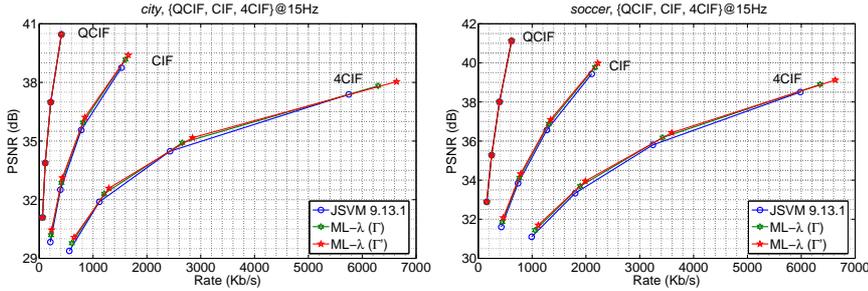
Table 1 (right part) presents the simulation results for spatial scalability where  $\Delta P_n$  and  $\Delta P_S$  represent the gain for the layer  $n$  and the sum gains for the three layers, respectively.



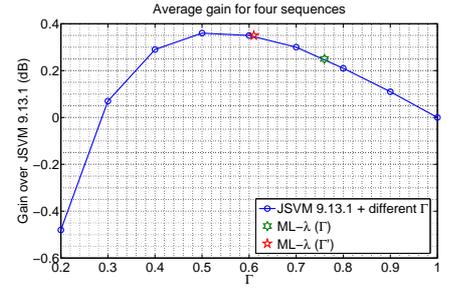
**Fig. 2.** Simulation results for quality scalability (four quality layers where  $QP = (36, 32, 28, 24)$ )



**Fig. 3.** Performance of different  $\Gamma$  values in quality scalability



**Fig. 4.** Simulation results for spatial scalability (three spatial layers where  $QP_0 = (34, 30, 26, 22)$ ,  $QP_1 = (36, 32, 28, 24)$ ,  $QP_2 = (38, 34, 30, 26)$ )



**Fig. 5.** Performance of different  $\Gamma$  values in Spatial Scalability

Since the layer 0 is identically coded with JSVM,  $\Delta P_0$  is always zero and is not listed. In addition, Fig. 4 shows two related R-D curves. On average, 0.25 dB and 0.35 dB gains over JSVM 9.13.1 are obtained by ML- $\lambda$  ( $\Gamma$ ) and ML- $\lambda$  ( $\Gamma'$ ), respectively.

Fig. 5 shows relationship between  $\Gamma$  value and the average gain  $\Delta P_S$ . Again, the best performance is obtained around the theoretical  $\Gamma'$  ( $\Gamma' = 0.61$ ), which indicates the proposed theory works well in spatial scalable scenario.

#### 4. CONCLUSIONS AND FUTURE WORK

In this paper, a Lagrange multiplier selection method ML- $\lambda$  is proposed for the rate-distortion optimization in SVC. By joint consideration of base and enhancement layers, a new factor  $\Gamma$  was introduced as a supplementary multiplier to the single layer  $\lambda$ . Compared with the current algorithm in recent SVC reference software, the new algorithm achieves a better coding efficiency at a negligible computational cost. Simulations shows that an average gain of 0.22 dB is obtained for a four-layer quality scalable case while 0.35 dB on average is gained for a three-layer spatial scalable environment. For the next step, a sequence adaptive  $\lambda$  selection method for SVC is to be considered. In addition, extending this work to an environment of combined scalability (including temporal scalability) is also an interesting topic.

#### 5. ACKNOWLEDGMENT

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