On Construction of Moderate-Length LDPC Codes over Correlated Erasure Channels

Gianluigi Liva, Balazs Matuz and Zoltán Katona
Institute of Communication and Navigation
Deutsches Zentrum fur Luft- und Raumfahrt (DLR)
82234 Wessling, Germany
Gianluigi.Liva@dlr.de

Enrico Paolini and Marco Chiani
DEIS, WiLAB
University of Bologna
via Venezia 52, 47023 Cesena (FC), Italy
{e.paolini,marco.chiani}@unibo.it

***Abstract*—The design of moderate-length erasure correcting low-density parity-check (LDPC) codes over correlated erasure channels is considered. Although the asymptotic LDPC code design remains the same as for a memoryless erasure channel, robustness to the channel correlation shall be guaranteed for the finite length LDPC code. This further requirement is of great importance in several wireless communication scenarios where packet erasure correcting codes represent a simple countermeasure for correlated fade events (e.g., in mobile wireless broadcasting services) and where the channel coherence time is often comparable with the code length. In this paper, the maximum tolerable erasure burst length (MTBL) is adopted as a simple metric for measuring the code robustness to the channel correlation. Correspondingly, a further step in the code construction is suggested, consisting of improving the LDPC code MTBL. Numerical results conducted over a Gilbert erasure channel, under both iterative and maximum likelihood decoding, highlight both the importance of the MTBL improvement in the finite-length code construction and the possibility to tightly approach the performance of maximum distance separable codes.**

I. Introduction

Recently, a large amount of research has been dedicated to capacity-approaching codes for the erasure channel (e.g., [1], [2]). One of the main practical applications of this theoretical investigation is represented by packet erasure recovery in wireless communication systems [3], [4]. Here, transfer frames are usually protected at the physical layer by bit-oriented error correcting codes. Moreover, an outer error detection code (cyclic redundancy check) is usually adopted to reveal corrupted frames, which are typically discarded. Hence, the upper layers of the protocol stack perceive either correctly received or erased data units, where erasures are typically correlated due to the channel fading process.

Lost data units can be recovered either by automatic retransmission query (ARQ) protocols or by packet erasure correction coding implemented at the upper layers of the communication stack. This latter solution is appealing especially in mobile wireless broadcasting services, as it avoids the need to handle retransmission requests from several users. Among erasure correcting codes, low-density parity-check (LDPC) codes [5] and Raptor codes [6] have recently gained an increasing attention due to both low implementation complexity and capacity-approaching performance. In a packet erasure correcting code, each information or encoded symbol represents a constant-length packet of bits.[1] For example, in a packet erasure correcting LDPC code each variable node (VN) represents a packet of bits and each check node (CN) represents a bit-wise parity constraint between the VNs connected to it.

It is well-known that a correlated erasure channel offers no advantages w.r.t. the memoryless erasure channel from a capacity viewpoint, as the reliability of the channel output can be determined by the channel output itself. Therefore, the asymptotic design of LDPC code ensembles for correlated erasure channels remains the same as for the memoryless erasure channel. This reasoning, however, breaks down when decoding is affected by finite-length restrictions.

A simple example is represented by a channel introducing one burst of erasures of length L per codeword, versus a channel introducing L erasures per codeword in random positions. In the bursty case, taking care that the code is capable to correct all erasure patterns of L contiguous positions guarantees a zero residual erasure rate. Under iterative (IT) LDPC decoding, this is equivalent to requiring that any set of L contiguous VNs contains no stopping sets. Under maximum likelihood (ML) decoding, this is equivalent to requiring that any L contiguous columns of the parity-check matrix are linearly independent. These tasks can be accomplished through a *judicious* permutation of the VNs (while effectiveness of a simple random permutation is not guaranteed). Note that under IT (resp. ML) decoding, this is in principle possible even if the minimum stopping set size (resp. minimum distance) is smaller than or equal to L. On the other hand, there is no way to guarantee a zero residual erasure rate under IT (resp. ML) decoding in the random erasure case, if the minimum stopping set size (resp. minimum distance) is smaller than or equal to L.

As illustrated by this simple example, a common way to face erasure correlation over burst erasure channels, within the framework of finite-length LDPC codes, consists of properly permuting the parity-check matrix columns to increase the code robustness to burst events (e.g., [7]–[10]). The robustness to a single erasure burst is measured by the maximum tolerable

[1] By *symbol* we denote either a bit or a packet of bits. In this paper we will assume a bit-oriented perspective, the extension to packets of bits being straightforward.

burst length (MTBL), denoted next by $L_{\max}$. For a given code, a given parity-check matrix representation, and a given decoding algorithm, the MTBL is the maximum length of an erasure burst which is correctable independently of its position within the codeword. For an (n, k) linear block code, $L_{\max}$ cannot exceed $n - k$.

In this paper, the construction of moderate-length erasure correcting LDPC codes over correlated erasure channels is considered. As the target application is represented by packet erasure recovery in mobile wireless broadcasting services, the two-state Gilbert erasure channel [11] is used to model data unit losses [12]–[14]. Concerning erasure recovery algorithm, both ML and IT LDPC decoding are considered. It is worth pointing out that ML erasure recovery represents a realistic option, as for LDPC codes over erasure channels it can be performed efficiently by exploiting the parity-check matrix sparseness [15], [16]. (For instance, ML decoding of Raptor codes is recommended for the UMTS MBMS service, where the decoder is implemented on a mobile terminal.[2])

The main contribution of this work is the development of a simple and practical LDPC coding scheme fulfilling the main requirements expected for a packet erasure correcting code:

- Systematic encoding.
- Low complexity encoding.
- Affordable ML decoding complexity.
- Performance under ML decoding close to that of an idealized maximum distance separable (MDS) code.[3]
- Reduced performance loss when switching to from ML to IT decoding.

A systematic encoding allows the received information to be directly delivered to the application, even upon a decoding failure. An affordable ML decoding complexity is mandatory as decoding may be performed in mobile terminals. Moreover, the possibility to switch to low-complexity IT decoding with a small performance penalty shall be guaranteed for terminals with severe limitations in processing capabilities.

The proposed coding scheme, achieving the above-mentioned requirements, is based on Generalized Irregular Repeat-Accumulate (GeIRA) codes [17]. The paper illustrates through numerical simulation how strengthening the finite-length code w.r.t. single erasure bursts, namely, increasing the code $L_{\max}$, can be an effective approach to achieve an ML decoding performance close to that of an idealized MDS code, while keeping a reduced performance penalty when switching to IT decoding.

II. Gilbert Erasure Channel

The two-state Gilbert erasure channel model is depicted in Fig. 1. It is based on a discrete-time Markov chain with one good state (G) and one bad state (B). State transitions are in correspondence with the transmission of each symbol, where the transition probabilities are p_{GG}, p_{GB}, p_{BG} and p_{BB}. The probabilities of receiving an erased symbol when the channel is in the state G and B are $e_G = 0$ and $e_B = 1$, respectively.

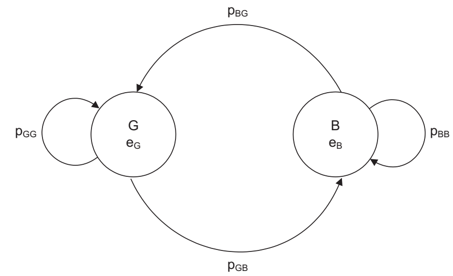

Fig. 1. Two-state Gilbert channel model

The stationary probabilities that the channel is in the state G and B are denoted by P_B and P_G, respectively. Note that P_B coincides with the average channel erasure probability ϵ. Introducing the average permanence in the state B, namely $\Delta_B \triangleq 1/p_{BG}$, the channel is completely specified by ϵ and Δ_B as $p_{BB} = 1 - 1/\Delta_B$ and $p_{GB} = \epsilon/[(1-\epsilon)\Delta_B]$. Note that Δ_B corresponds to the average erasure burst length, and represents a measure of the channel correlation.

A. Singleton Bound

Let us denote by $P(n, i)$ the probability of having i erasures in n consecutive symbols. Since a linear block code can always recover from any pattern of $d_{\min} - 1$ or less erasures, and since it cannot recover from any pattern of $n - k + 1$ or more erasures, the probability of decoding failure of an idealized (n, k) MDS code under ML decoding is given by

$$P_{e,id} = \sum_{i=n-k+1}^{n} P(n, i) \,. \tag{1}$$

Note that, for any (n, k) linear block code, we have $P_e \geq P_{e,id}$, which is referred to as the Singleton bound. Over a Gilbert erasure channel, the RHS of (1) can be computed as sketched next. For each i we have

$$P(n, i) = P(n, i \,|\, \zeta)\, P_G + P(n, i \,|\, \overline{\zeta})\, P_B \tag{2}$$

having denoted by ζ (resp. $\overline{\zeta}$) the event that the channel is in the state G (resp. B) during the transmission of the first symbol. The calculation of $P(n, i \,|\, \zeta)$ and $P(n, i \,|\, \overline{\zeta})$ can be performed using recursive formulas developed in [18].

III. Code Construction and Decoding

A. Efficient ML Decoding over the Erasure Channel

Let us consider an (n, k) binary linear block code with parity-check matrix $\mathbf{H}$. Over an erasure channel, ML decoding is equivalent to solving the linear equation

$$\mathbf{x}_{\overline{K}}\, \mathbf{H}_{\overline{K}}^T = \mathbf{x}_K\, \mathbf{H}_K^T \,. \tag{3}$$

In (3) $\mathbf{x}_{\overline{K}}$ and $\mathbf{x}_K$ denote the set of erased and correctly received encoded bits, respectively. Analogously, $\mathbf{H}_{\overline{K}}$ and

[2] In the case of Raptor codes the sparseness of the *constraint matrix* (instead of the parity-check matrix) is exploited, but the principle remains the same.

[3] The minimum distance $d_{\min}$ of any linear block code satisfies $d_{\min} \leq n - k + 1$. Throughout the paper, we call *idealized MDS code* an (n, k) binary linear block code achieving this bound with equality. Note that, in general, this code does not exist in the binary case.

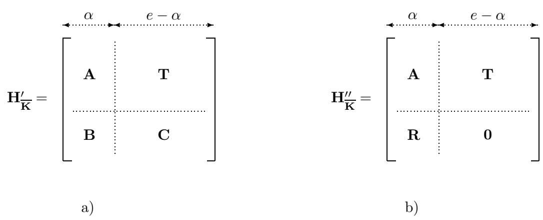

Fig. 2. Steps for reduced complexity ML decoding.

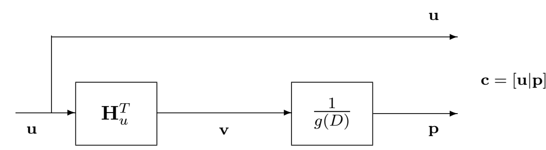

Fig. 3. GeIRA encoder. Decomposing the parity-check matrix as $\mathbf{H} = [\mathbf{H}_u|\mathbf{H}_p]$, $\mathbf{H}_u$ corresponding to the k systematic symbols and $\mathbf{H}_p$ to the $m = n - k$ parity symbols, we have that $\mathbf{H}_u^T$ is the outer LDGM code generator matrix. Moreover, $\mathbf{H}_p$ is specified by the feedback polynomial $g(D) = \sum_{j=0}^{t} g_j D^j$ of the inner rate-1 RCC (where $g_j \in \{0, 1\}$ and $g_0 = g_t = 1$).

$\mathbf{H}_K$ denote the submatrices composed of the $\mathbf{H}$ columns corresponding to $\mathbf{x}_{\overline{K}}$ and $\mathbf{x}_K$, respectively. Equation (3) can be solved by Gaussian elimination (GE) on $\mathbf{H}_{\overline{K}}$ with complexity $O(n^3)$. As n increases, ML decoding may become impractical.

For moderate block lengths, (3) can be solved through a reduced complexity approach [15], [16] exploiting the sparseness of the parity-check matrix.[4] Provided the number of erasures $e = |\mathbf{x}_{\overline{K}}|$ does not exceed $n-k$, $\mathbf{H}_{\overline{K}}$ is first reduced to an approximate triangular matrix $\mathbf{H}'_{\overline{K}}$, as depicted in Fig. 2(a) (where $\mathbf{T}$ is an $((e - \alpha) \times (e - \alpha))$ lower triangular matrix), through row / column permutations only. In a second step, $\mathbf{C}$ is made equal to the zero matrix by row additions, leading to $\mathbf{H}''_{\overline{K}}$ in Fig. 2(b). Finally, GE is applied to $\mathbf{R}$, thus recovering the α leftmost unknowns (called the *reference symbols*). The remaining $e - \alpha$ unknowns are recovered by simple back-substitution. A decoding failure takes place whenever the rank of $\mathbf{R}$ is smaller than α. Note that the performance of the reduced complexity ML decoder is the same as that of full GE.

As the complexity is dominated by the GE step applied to $\mathbf{R}$, a small α is mandatory. It is illustrated in [16] how a small α is associated with a large IT decoding threshold ϵ^*_{IT}.

B. GeIRA Codes

The adopted coding scheme is based on GeIRA codes [17]. They are systematic LDPC codes generating the parity bits by a serial concatenation of an outer low-density generator matrix (LDGM) code with an inner rate-1 recursive convolutional code (RCC), as depicted in Fig. 3.

The class of GeIRA codes is considered in this paper as, besides efficient ($O(n)$) and systematic encoding, they offer a large flexibility in terms of degree distribution selection, which allows trading off the several requirements recalled in Section I. Properly adjusting the distribution of the $\mathbf{H}_u$ matrix and the feedback polynomial $g(D)$, it is possible to design GeIRA ensembles characterized by an asymptotic ML decoding threshold ϵ^*_{ML} [19] very close to capacity, and a good compromise between a large IT decoding threshold ϵ^*_{IT} (useful to reduce the number of reference symbols and to obtain a small performance gap in the waterfall region when switching from ML to IT decoding) and minimum stopping set size / minimum distance.

As recalled in Section I, a judicious permutation of the parity-check matrix columns can be very effective in improving the performance of finite-length LDPC codes over correlated erasure channels. The metric here adopted to measure the code capability to exploit erasure correlation is the MTBL. An effective algorithm, proposed in [9], to improve the MTBL of a given LDPC code is reviewed next.

C. Brief Review of an $L_{\max}$ Improving Algorithm

The algorithm is based on the concept of *stopping set pivot*. Given a stopping set $\mathcal{S}$, let $\mathcal{G}$ be its induced subgraph. A VN $V \in \mathcal{S}$ is said a pivot of $\mathcal{S}$ when the following condition is fulfilled: if the value of V is known and the value of all the other VNs of $\mathcal{G}$ is unknown, then the IT decoder applied to $\mathcal{G}$ recovers successfully from the erasure pattern. Any non-empty stopping set has at least two pivots [9, Theorem 1]. Moreover, if an erasure burst of length $L+1$ is uncorrectable for an LDPC code with $L_{\max} = L$, then the first and the last symbols of the burst are pivots of the associated maximal stopping set.

The algorithm increases the span of the pivots of all the stopping sets by column permutations, to make single erasure bursts (of length a large as possible) correctable under IT decoding. To this aim, given an LDPC code with $L_{\max} = L$, all the uncorrectable erasure bursts of length $L + 1$ are identified and, for each burst, some pivots of the associated stopping set are found (*pivot searching step*). Next, one pivot of each burst is selected and swapped with a VN not in the burst (*pivot swapping step*). If the pivot swapping makes the burst length $L+1$ correctable for all burst positions, then $L_{\max}$ is set to $L+1$ and the burst length $L+2$ is considered. Otherwise, a different pivot swapping is performed. The algorithm complexity for increasing the MTBL from L to $L+1$ is $O(n^2)$.

The algorithm improves the MTBL under IT decoding of any given finite-length LDPC code. As the ML MTBL is lower bounded by the IT MTBL, the algorithm is also effective for ML decoding.[5] Note that applying the algorithm to a GeIRA code would jeopardize the efficient encoder depicted in Fig. 3. In this case, the algorithm shall be considered as a tool for the design of an *ad hoc* interleaver placed after the encoder

[4] It is worth observing that ML decoding over a binary erasure channel can be also performed using a bit guessing approach [19] This approach, however, is not practical over packet erasure channels, as in this case all the bits composing an erased packet should be guessed.

[5] Even though L columns of the parity-check matrix are linearly independent, they may contain stopping sets. Therefore, even if the algorithm could be targeted on ML decoding instead of IT decoding (i.e., on independent sets instead of stopping sets), that would lead to a loose of control of the IT MTBL, which may be even worsened by the algorithm.

TABLE I
VALUES OF $L_{\max}$ FOR GEIRA CODES. THE EFFICIENCY η IS DEFINED AS $\eta \triangleq L_{\max}/(n-k)$.

GeIRA (512,256)				
Permutation	IT $L_{\max}$	ML $L_{\max}$	η_{IT}	η_{ML}
-	30	207	0.12	0.81
$\pi_{i,opt}$	177	207	0.69	0.81
π_{opt}	221	246	0.86	0.96

GeIRA (2048,1024)				
Permutation	MTBL IT	MTBL ML	η_{IT}	η_{ML}
-	138	701	0.13	0.68
$\pi_{i,opt}$	730	740	0.71	0.72
π_{opt}	904	1013	0.88	0.99

and operating on the codeword $\mathbf{c} = [\mathbf{u}|\mathbf{p}]$. To avoid the extra interleaving step, one may apply the algorithm to the systematic VNs only. With this approach the encoder structure remains that of Fig. 3, with properly permuted columns of $\mathbf{H}_u$. The permutation applied to all the VNs is denoted by π_{opt}, while the permutation applied to the systematic VNs by $\pi_{i,opt}$.

D. Code Design and Construction

As pointed out in Section I, the asymptotic ensemble design of LDPC codes over correlated erasure channels remains essentially the same as for the memoryless erasure channel. For this reason, we consider a $R = 1/2$ GeIRA ensemble that was selected in [20] for the memoryless channel. It is characterized by a regular CN degree 9, a feedback polynomial of the rate-1 RCC $g(D) = 1 + D + D^{\lfloor 0.24\,n \rfloor}$ and a systematic VN distribution from a node perspective given by $\Lambda(x) = 0.7813x^3 + 0.1914x^{49} + 0.0195x^{53} + 0.0078x^{54}$ (the coefficient of x^i is the fraction of systematic VNs of degree i). The thresholds are $\epsilon^*_{\mathrm{IT}} = 0.480$ and $\epsilon^*_{\mathrm{ML}} = 0.497$.

We constructed a $(512, 256)$ and a $(2048, 1024)$ GeIRA codes from this ensemble. The adopted construction method consisted of: 1) generating $\mathbf{H}_p$ according to $g(D)$; 2) generating $\mathbf{H}_u$ using the progressive edge-growth (PEG) algorithm [21] (taking into account the 1s already placed in $\mathbf{H}_p$); 3) improving the MTBL by using the previously reviewed algorithm. The MTBL values of the two GeIRA codes under IT and ML decoding at the end of step 2 are shown in Table I together with the MTBL values obtained by applying both the full permutation (π_{opt}) and the systematic permutation ($\pi_{i,opt}$). Remarkably, the π_{opt}-codes achieve a very large IT MTBL (221 and 904) and a close-to-optimum ML MTBL (246 and 1013). The $\pi_{i,opt}$ permutation, though leading to an IT MTBL improvement (177 vs. 30 and 730 vs. 138), turned out to be less effective under ML decoding.

IV. NUMERICAL RESULTS

Monte Carlo simulations of the codes described in Subsection III-D were performed on the Gilbert erasure channel, collecting 500 decoding failures for each point.

Fig. 4 shows the codeword error rate (CER) vs. the average channel erasure probability ϵ for the $(2048, 1024)$ codes of Table I, assuming a highly correlated channel with $\Delta_B = 100$. Remarkably, the larger $L_{\max}$ in Table I, the smaller the

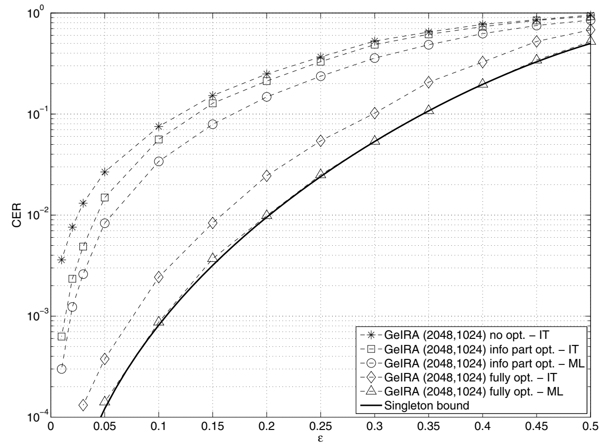

Fig. 4. Codeword error rate as vs. the average erasure rate, for various permutations of the $(2048, 1024)$ GeIRA code ($\Delta_B = 100$).

coding gain loss w.r.t. the Singleton bound. For instance, the π_{opt}-$(2048, 1024)$ code under IT decoding has a better MTBL, and correspondingly a better performance, than the $\pi_{i,opt}$-$(2048, 1024)$ code under ML decoding. This confirm the effectiveness of the $L_{\max}$ metric.

Note that the π_{opt}-$(2048, 1024)$ code approaches tightly the Singleton bound under ML decoding, down to low error rates, thanks to the combination of a capacity approaching ϵ^*_{ML}, a PEG-based code construction technique reducing the number of small size stopping sets, and a judicious symbol interleaving (π_{opt}). Furthermore, thanks to a good ϵ^*_{IT}, the number of reference symbols is kept small with a consequent affordable ML decoding complexity: a ML decoder implementation for this code works at 1.4 Gbits/s over a commercial PC processor in the worst erasure pattern conditions, where the IT decoder works at 4.0 Gbits/s [22]. The combination of a good ϵ^*_{IT} with a proper symbol interleaving leads to a performance of the π_{opt}-$(2048, 1024)$ code which remains satisfying if switching to low-complexity IT decoding.

The ML performance of the π_{opt}-$(2048, 1024)$ GeIRA code is presented in Fig. 5 over Gilbert erasure channels with different correlation levels Δ_B. The proposed code construction provides a nearly optimal performance in all cases.

Though it is affordable for moderate lengths, ML decoding may become impractical (even with a reduced complexity approach) for much larger lengths. A remedy is to rely on short codes in conjunction with block interleavers spanning over several codewords, as proposed in [14]. For instance, let us consider Fig. 6, where the ML performance of the π_{opt}-$(2048, 1024)$ code is compared with the performance of the π_{opt}-$(512, 256)$ code with interleaving depth 4. More specifically, the $4 \cdot 512$ block interleaver is filled row-wise with four $n = 512$ codewords, and then read column-wise. The overall latency due to the interleaver is the same as that of the π_{opt}-$(2048, 1024)$ interleaver. The plots show simulations on channels with different values of Δ_B. For highly correlated channels both approaches give the same results close to the

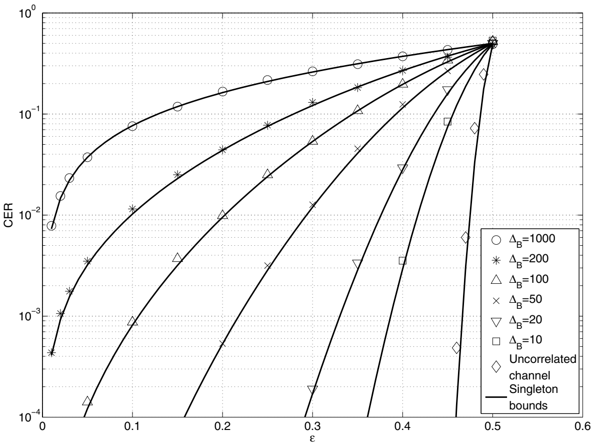

Fig. 5. Codeword error rate vs. the average erasure rate, for the π_{opt}-$(2048, 1024)$ code under ML decoding.

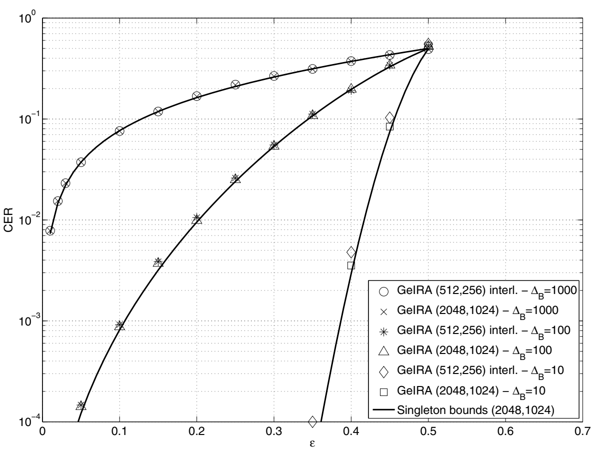

Fig. 6. Codeword error rate vs. the average erasure rate, for the π_{opt}-$(2048, 1024)$ code and for the block-interleaved π_{opt}-$(512, 256)$ GeIRA code (interleaving depth 4) under ML decoding.

Singleton bound.

V. Conclusion

A simple and effective construction method for moderate-length LDPC codes over correlated (Gilbert) erasure channels has been presented. The proposed method focuses on the class of the efficiently-encodable GeIRA codes. It leads to codes satisfying the main requirements of a packet erasure correcting code in mobile broadcasting scenarios, namely, systematic and efficient encoding, performance under ML decoding close to that of an idealized MDS code obtained with a manageable complexity, possibility to switch to low complexity IT decoding with a reduced performance penalty. It has been illustrated how the simple parameter $L_{\max}$ represents an effective performance metric over the Gilbert erasure channel. Its enhancement through column permutations represents an important code construction step to approach the performance of idealized codes under ML decoding while preserving a satisfying performance under low-complexity IT decoding.

Acknowledgment

The work was supported in part by the European Community under Seventh Framework Programme grant agreement ICT OPTIMIX nINFSO-ICT-214625 and by the EC-IST SatNEx-II project (IST-27393).

References

[1] M. Luby, M. Mitzenmacher, M. Shokrollahi, and D. Spielman, "Efficient erasure correcting codes," *IEEE Trans. Inform. Theory*, vol. 47, no. 2, pp. 569–584, Feb. 2001.

[2] P. Oswald and M. A. Shokrollahi, "Capacity-achieving sequences for the erasure channel," *IEEE Trans. Inform. Theory*, vol. 48, no. 12, pp. 364–373, Dec. 2002.

[3] G. Faria, J. Henriksson, E. Stare, and P. Talmola, "DVB-H: Digital broadcast services to handheld devices," *Proceedings of the IEEE*, vol. 94, no. 1, pp. 194–209, Jan. 2006.

[4] 3GPP TS 26.346 V6.1.0, "Technical specification group services and system aspects; multimedia broadcast/multicast service; protocols and codecs," June 2005.

[5] R. G. Gallager, *Low-Density Parity-Check Codes*. Cambridge, MA: M.I.T. Press, 1963.

[6] M. Shokrollahi, "Raptor codes," *IEEE Trans. Inform. Theory*, vol. 52, no. 6, pp. 2551–2567, June 2006.

[7] M. Yang and W. E. Ryan, "Design of LDPC codes for two-state fading channel models," in *Proc. of 5th Int. Symp. on Wireless Personal Multimedia Communications*, vol. 3, Oct. 2002, pp. 986–990.

[8] T. Wadayama, "Greedy construction of LDPC codes for burst erasure channel," *IEICE Tech. Report*, vol. 104, no. 302, pp. 35–40, Sept. 2004.

[9] E. Paolini and M. Chiani, "Construction of near-optimum burst erasure correcting LDPC codes," *IEEE Trans. Commun.*, Sept. 2008, to appear.

[10] M. Fossorier, "Hybrid burst erasure correction of LDPC codes," *IEEE Commun. Lett.*, to appear.

[11] E. N. Gilbert, "Capacity of a burst-noise channel," *Bell Syst. Tech. J.*, vol. 39, pp. 1253–1265, Sept. 1960.

[12] M. Zorzi, R. Rao, and L. Milstein, "Error statistics in data transmission over fading channels," *IEEE Trans. Commun.*, vol. 46, no. 11, pp. 1468–1477, Nov. 1998.

[13] E. Lutz, D. Cygan, M. Dippold, F. Dolainsky, and W. Papke, "The land mobile satellite communication channel — recording, statistics and channel model," *IEEE Trans. Vehicular Technology*, vol. 40, no. 2, pp. 375–386, May 1991.

[14] L. Wilhelmsson and L. B. Milstein, "On the effect of imperfect interleaving for the Gilbert–Elliott channel," *IEEE Trans. Commun.*, vol. 47, no. 5, pp. 681–688, May 1999.

[15] B. A. LaMacchia and A. M. Odlyzko, "Solving large sparse linear systems over finite fields," *Lecture Notes in Computer Science*, vol. 537, pp. 109–133, 1991.

[16] D. Burshtein and G. Miller, "An efficient maximum likelihood decoding of LDPC codes over the binary erasure channel," *IEEE Trans. Inform. Theory*, vol. 50, no. 11, pp. 2837–2844, nov 2004.

[17] G. Liva, E. Paolini, and M. Chiani, "Simple reconfigurable low-density parity-check codes," *IEEE Commun. Lett.*, vol. 9, no. 3, pp. 258–260, Mar. 2005.

[18] E. O. Elliott, "Estimates of error rates for codes on burst-noise channels," *Bell Syst. Tech. J.*, vol. 42, pp. 1977–1997, 1963.

[19] C. Measson, A. Montanari, and R. Urbanke, "Maxwell's construction: the hidden bridge between iterative and maximum a posteriori decoding," *IEEE Trans. Inform. Theory*, vol. 54, no. 12, pp. 5277–5307, Dec. 2008.

[20] E. Paolini, G. Liva, B. Matuz, and M. Chiani, "Generalized IRA erasure correcting codes for hybrid iterative / maximum likelihood decoding," *IEEE Commun. Lett.*, vol. 12, no. 6, pp. 450–452, June 2008.

[21] X. Hu, E. Eleftheriou, and D. Arnold, "Regular and irregular progressive edge-growth Tanner graphs," *IEEE Trans. Inform. Theory*, vol. 51, no. 1, pp. 386–398, Jan. 2005.

[22] G. Liva, E. Paolini, and M. Chiani, "Packet loss recovery in the telemetry downlink via maximum-likelihood LDPC decoding," in *Proc. of the 10th Int. Workshop on Signal Processing for Space Communications*, Rhodes Island, Greece, Oct. 2008, pp. 1–6.