

# Optimized Simple Bounds for Diversity Systems

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**Abstract**—Diversity techniques play a key role in modern wireless systems, whose design benefits from a clear understanding of how these techniques affect system performance. To this aim we propose a simple class of bounds, whose parameters are optimized, on the symbol error probability (SEP) for detection of arbitrary two-dimensional signaling constellations with diversity in the presence of non-ideal channel estimation. Unlike known bounds, the optimized simple bounds are tight for *all* signal-to-noise ratios (SNRs) of interest. In addition, these bounds are easily invertible, which enables us to obtain bounds on the symbol error outage (SEO) and SNR penalty. As example applications for digital mobile radio, we consider the SEO in log-normal shadowing and the SNR penalty for both maximal ratio diversity, in the case of unequal branch power profile, and subset diversity, in the case of equal branch power profile, with non-ideal channel estimation. The reported lower and upper bounds are extremely tight, that is, within a fraction of a dB from each other.

**Index Terms**—Performance evaluation, optimized simple bounds, multichannel reception, fading channels, non-ideal channel estimation, inverse symbol error probability.

## I. INTRODUCTION

A CLEAR understanding of how diversity techniques affect system performance is important for the design of modern wireless systems. Recently, such techniques have been proposed to improve the performance of third generation (3G) and beyond 3G wireless networks (see, e.g., [1]–[4]). These networks typically operate in situations where the received signals are sufficiently spaced or delayed such that in the presence of both small- and large-scale fading the received branch signal-to-noise ratios (SNRs) are independent non-identically distributed (INID) random variables (r.v.'s). Specific cases include: 1) angle diversity using multiple beams, where the average received signal strength can be different for each beam; 2) polarization diversity using horizontal and vertical polarization with high base station antennas, where, for a vertically-polarized antenna, the average received signal strength is typically 6 to 10 dB lower than that for a

horizontally-polarized antenna; 3) macrodiversity, where each channel is subject to different path-loss and shadowing; and 4) Rake receivers operating in environments with unequal power dispersion profile.

The performance of diversity systems in terms of symbol and bit error probability (BEP) averaged over small-scale fading has been studied extensively in the literature, with direct applications to antenna diversity and Rake reception [5]–[13]. In particular, closed-form expressions for the average BEP of binary phase shift keying (PSK) with coherent detection and maximal-ratio combining (MRC), are given in [13], and the symbol error probability (SEP) of  $M$ -ary PSK is discussed in [12]–[14]. Although it is possible in some cases to write a closed-form expression, the alternative expression for the SEP obtained by either the characteristic function [15]–[18], or the equivalent moment generating function (MGF) method [12] displays the dependence on the SNR and diversity technique. More recently, diversity techniques in the presence of non-ideal channel estimation have received increased attention [19]–[32]. In fact, practical systems must first estimate the channel on each diversity branch, then combine the signals on the branches using weights that depend on these estimates and the combining technique. Consequently, due to imperfect estimation, systems incur a performance loss which depends on the trade-off between the amount of energy dedicated to data transmission and channel estimation (see, e.g., [27]–[29]).

When receiver complexity constrains the use of all the available branches in space or time, subset diversity (SSD) can be utilized [33]–[35]. SSD is a method by which a subset of the available signals from the branches are selected and then combined. Such systems, although they only make use of a subset of the available branches, are capable of achieving significant performance improvement over a single branch receiver [36]–[44]. As an example, in hybrid-selection/MRC (H-S/MRC), the  $L$  out of  $N$  branches with the strongest signals are selected and then combined. In the case of SSD, channel estimation plays a *dual role*. Specifically, the chosen subset of diversity branches is based on knowledge of the channel, i.e., the *estimated* channel gains. These estimates are then used to weight the branches during the combining process. Thus, channel estimation affects both the selection process, as well as the combining mechanism [28].

In many important problems related to wireless mobile communication, explicit expressions for the inverse SEP, that is, SNR as a function of the target SEP, are also required [45], [46]. One example is the outage probability defined as the probability that the SEP exceeds a maximum tolerable level. We shall refer to this quality of service (QoS) measure as symbol error outage (SEO). This definition of SEO is appro-

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appropriate for digital communication systems and its evaluation requires such an inverse SEP expression [45]–[48]. Inverse SEP expressions are also useful when evaluating the SNR penalty between different systems at a given target SEP [38], [49], [50]. However, it is not straightforward to obtain the inverse SEP. Although closed-form SEP expressions can be found in some special cases, the inverse SEP function does not exist in general, especially in the presence of channel estimation errors. Therefore, inversion of the SEP typically requires numerical root evaluation. To make problems of this nature analytically tractable, we propose to replace the exact SEP with bounds that are easily invertible and tight for *all* values of SNR.

An important analysis that allows a quick assessment of the SEP behavior is the asymptotic bound (see, e.g., [13], [51]). However, numerical investigations reveal that for the SEPs of interest (i.e., in the range  $10^{-3}$  to  $10^{-1}$  for uncoded systems) at low and moderate SNR, the asymptotic bounds are quite loose, especially as the number of diversity branches increases. To address this problem, uniformly tight and invertible bounds were reported in [45] for independent identically distributed (IID) Rayleigh fading channels. A general analysis of the behavior of the BEP in Gaussian noise for multidimensional signaling constellations and various fading statistics is given in [52].

In this paper we derive new easily invertible upper and lower bounds on the SEP with parameters optimized within a given class. These bounds, referred to as optimized simple bounds (OSBs), are applicable to systems employing arbitrary two-dimensional signaling constellations and diversity techniques. We consider the widely-used Rayleigh fading channel model superimposed on log-normal shadowing. We examine non-ideal channel estimation in conjunction with: 1) maximal ratio diversity in the case of unequal branch power profile (INID diversity branches); and 2) subset diversity in the case of equal branch power profile (IID diversity branches). Thus, the performance of practical digital wireless communication systems can be easily characterized, in terms of the SEP, SEO, and SNR penalty, using rigorous lower and upper bounds.

The remainder of this paper is organized as follows. In Section II we state the general problem and describe the new class of bounds. We then provide some insights on known bounds in the literature and derive optimal bounds within this class. In Section III we apply our optimized simple bounds to problems including the evaluation of SEP, SEO, and SNR penalty. In Section IV we provide numerical results, and we give our conclusions in Section V.

## II. A NEW CLASS OF BOUNDS

In this section we first discuss the problem and then describe the new class of bounds. Then we derive the optimal bounds within this class.

### A. Preliminaries

In several important scenarios, the SEP is given by a convex combination of terms of the form

$$I_N(\zeta, \phi, \psi) \triangleq \frac{1}{2\pi} \int_0^\phi \prod_{n=1}^N \frac{\sin^2(\theta + \psi)}{\sin^2(\theta + \psi) + \zeta_n} d\theta, \quad (1)$$

where the vector  $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_N]$ , and  $\zeta_n$  is a function of the  $n^{\text{th}}$  branch SNR.<sup>1</sup> Thus, finding upper and lower bounds on the SEP essentially reduces to finding upper and lower bounds on  $I_N(\zeta, \phi, \psi)$ .

By noting that  $0 \leq \sin^2(\theta + \psi) \leq 1$ , one can immediately obtain upper and lower bounds on  $I_N(\zeta, \phi, \psi)$ . In fact, by substituting  $\sin^2(\theta + \psi)$  with its minimum value (i.e., 0) in the denominator of the integrand in (1), we immediately obtain an upper-bound:

$$I_N(\zeta, \phi, \psi) \leq \frac{S_N(\phi, \psi)}{\prod_{n=1}^N \zeta_n}, \quad (2)$$

where

$$S_N(\phi, \psi) \triangleq \frac{1}{2\pi} \int_0^\phi \sin^{2N}(\theta + \psi) d\theta. \quad (3)$$

Similarly, we can also obtain a lower bound on  $I_N(\zeta, \phi, \psi)$  by replacing  $\sin^2(\theta + \psi)$  with its maximum value (i.e., 1), in the denominator of the integrand in (1), and we obtain

$$\frac{S_N(\phi, \psi)}{\prod_{n=1}^N (1 + \zeta_n)} \leq I_N(\zeta, \phi, \psi). \quad (4)$$

Unfortunately, as will be shown in Sec. IV, for low and moderate values of the  $\zeta_n$ 's, the bounds in (2) and (4) depart from the exact expression (1) as  $N$  increases. In the following subsection, we propose bounds to overcome this problem.

### B. Optimized Simple Bounds: The Key Idea

The key observation is that for low values of  $\zeta_n$ , the contribution of  $\sin^2(\theta + \psi)$  in the denominator of the integrand in (1) is not negligible. Since our goal is to obtain lower and upper bounds that are tight for *all* values of the  $\zeta_n$ 's, we propose the following class of bounds for  $I_N$ :

$$I_{N,L}(\zeta, \phi, \psi) \leq I_N(\zeta, \phi, \psi) \leq I_{N,U}(\zeta, \phi, \psi), \quad (5)$$

where  $I_{N,L}(\zeta, \phi, \psi)$  and  $I_{N,U}(\zeta, \phi, \psi)$  have the following form:

$$I_{N,L}(\zeta, \phi, \psi) = \frac{S_N(\phi, \psi)}{\prod_{n=1}^N [C_{N,L}(\phi, \psi) + \zeta_n]}, \quad (6a)$$

$$I_{N,U}(\zeta, \phi, \psi) = \frac{S_N(\phi, \psi)}{\prod_{n=1}^N [C_{N,U}(\phi, \psi) + \zeta_n]}, \quad (6b)$$

with  $0 \leq C_{N,U}(\phi, \psi) \leq C_{N,L}(\phi, \psi) \leq 1$  independent of  $\zeta$ . Note that (2) and (4) belong to this class with  $C_{N,U}(\phi, \psi) = 0$  and  $C_{N,L}(\phi, \psi) = 1$ .

Our goal is to find the optimal  $C_{N,L}(\phi, \psi)$  and  $C_{N,U}(\phi, \psi)$  such that (5) is valid for all values of  $\zeta$ . With this in mind, we first define the function  $C_N(\zeta, \phi, \psi)$  implicitly as follows:

$$I_N(\zeta, \phi, \psi) = \frac{1}{2\pi} \int_0^\phi \prod_{n=1}^N \frac{\sin^2(\theta + \psi)}{\sin^2(\theta + \psi) + \zeta_n} d\theta \triangleq \frac{S_N(\phi, \psi)}{\prod_{n=1}^N [C_N(\zeta, \phi, \psi) + \zeta_n]}. \quad (7)$$

<sup>1</sup>For several cases of interest,  $\zeta_n$  increases monotonically with the  $n^{\text{th}}$  branch SNR (see Table II). Unless otherwise stated, the terms SNR and SEP are used to denote the mean SNR and the mean SEP, respectively, averaged over the small-scale fading.

Note that the function  $C_N(\zeta, \phi, \psi)$  is well defined and unique for each  $\zeta$ , since the mapping

$$h(x) : \mathbb{R}^+ \rightarrow (0, h_0],$$

defined by

$$h(x) \triangleq \frac{S_N(\phi, \psi)}{\prod_{n=1}^N (x + \zeta_n)}, \quad x \in \mathbb{R}^+, \quad (8)$$

with  $h_0 = h(0)$ , is a continuous and strictly decreasing function of  $x$ .<sup>2</sup> Also note that from (2) and (4),  $I_N(\zeta, \phi, \psi) \in [h(1), h(0)]$ . Moreover, since  $I_N(\zeta, \phi, \psi)$  is a continuous function of each  $\zeta_n$ ,  $C_N(\zeta, \phi, \psi)$  is also continuous in each  $\zeta_n$ . We will study the behavior of  $C_N(\zeta, \phi, \psi)$  in the following.<sup>3</sup>

### C. Behavior of $C_N(\zeta, \phi, \psi)$

**Theorem 1:** [Monotonicity of  $C_N(\zeta, \phi, \psi)$ ] The function  $C_N(\zeta, \phi, \psi)$  is monotonically increasing in  $\zeta_n$ , for each  $n$ .

*Proof:* Without loss of generality we will show that  $C_N(\zeta, \phi, \psi)$  is monotonically increasing in  $\zeta_1$ . Let us consider an increment of  $\zeta_1$  and define a new  $\tilde{\zeta} = [\tilde{\zeta}_1, \tilde{\zeta}_2, \dots, \tilde{\zeta}_N]$  where

$$\tilde{\zeta}_n = \begin{cases} \zeta_1 + \Delta\zeta_1 & n = 1, \\ \zeta_n & \text{otherwise.} \end{cases} \quad (9)$$

Next, we verify that the variation in  $C_N(\zeta, \phi, \psi)$ , that is

$$\Delta C = C_N(\tilde{\zeta}, \phi, \psi) - C_N(\zeta, \phi, \psi),$$

as a function of  $\Delta\zeta_1$  is non-negative for all values of  $\zeta$  and  $\Delta\zeta_1 > 0$ . Note that  $I_N(\zeta, \phi, \psi)$  is continuous and strictly decreasing in  $\zeta_n$  for each  $n$ , and since the vector  $\tilde{\zeta}$  differs from the vector  $\zeta$  only in the first component, we have

$$I_N(\tilde{\zeta}, \phi, \psi) \leq I_N(\zeta, \phi, \psi) \quad (10)$$

The above inequality, together with (7), implies that

$$\begin{aligned} & \left[ C_N(\tilde{\zeta}, \phi, \psi) + (\zeta_1 + \Delta\zeta_1) \right] \prod_{n=2}^N \left[ C_N(\tilde{\zeta}, \phi, \psi) + \zeta_n \right] \\ & \geq \prod_{n=1}^N \left[ C_N(\zeta, \phi, \psi) + \zeta_n \right], \\ 1 & \leq \left[ 1 + \frac{\Delta C + \Delta\zeta_1}{C_N(\zeta, \phi, \psi) + \zeta_1} \right] \prod_{n=2}^N \left[ 1 + \frac{\Delta C}{C_N(\zeta, \phi, \psi) + \zeta_n} \right]. \end{aligned}$$

Taking the logarithm of both sides, gives

$$\begin{aligned} & \ln \left( 1 + \frac{\Delta C + \Delta\zeta_1}{C_N(\zeta, \phi, \psi) + \zeta_1} \right) \\ & + \sum_{n=2}^N \ln \left( 1 + \frac{\Delta C}{C_N(\zeta, \phi, \psi) + \zeta_n} \right) \geq 0. \end{aligned} \quad (11)$$

Now, we take the limit of (11) for  $\Delta\zeta_1 \rightarrow 0$ , with  $\Delta C \rightarrow \partial C$ , which results in

$$\sum_{n=1}^N \ln \left( 1 + \frac{\partial C}{C_N(\zeta, \phi, \psi) + \zeta_n} \right) \geq 0. \quad (12)$$

<sup>2</sup>When explicit definitions of functions are not possible, implicit definitions enable elegant solutions to mathematical problems. Without the use of an implicit definition, the solution to the problem at hand would have been very cumbersome.

<sup>3</sup>Problems akin to this were addressed in [53].

It is clear that  $\partial C \geq 0$ ; otherwise,  $\partial C < 0$  would imply that all the terms in the sum of (12) are negative, which is a contradiction. Similar arguments can be used to prove that the partial derivative of  $C_N(\zeta, \phi, \psi)$  with respect to each  $\zeta_n$  is also nonnegative.  $\square$

### D. Derivation of the Optimized Simple Bounds

By using the property above we arrive at the OSBs for  $I_N(\zeta, \phi, \psi)$ .

**Theorem 2:** [Optimized Simple Bounds for  $I_N(\zeta, \phi, \psi)$ ] The function  $I_N(\zeta, \phi, \psi)$  is lower and upper bounded by

$$I_{N,L}(\zeta, \phi, \psi) \leq I_N(\zeta, \phi, \psi) \leq I_{N,U}(\zeta, \phi, \psi), \quad (13)$$

where the optimal values for  $C_{N,L}(\phi, \psi)$  and  $C_{N,U}(\phi, \psi)$  are given by

$$C_{N,L}(\phi, \psi) = \frac{S_{N+1}(\phi, \psi)}{S_N(\phi, \psi)}, \quad (14a)$$

$$C_{N,U}(\phi, \psi) = \left[ \frac{2\pi}{\phi} S_N(\phi, \psi) \right]^{1/N}. \quad (14b)$$

*Proof:* The definition of  $C_N(\zeta, \phi, \psi)$  in (7) implies that any  $C_{N,L}(\phi, \psi)$  and  $C_{N,U}(\phi, \psi)$  satisfying

$$C_{N,L}(\phi, \psi) \geq C_N(\zeta, \phi, \psi), \quad (15)$$

$$C_{N,U}(\phi, \psi) \leq C_N(\zeta, \phi, \psi), \quad (16)$$

for all values of  $\zeta$ , provide us with bounds of the form (13). Since  $h(x)$  in (8) is strictly decreasing, the optimal  $C_{N,L}(\phi, \psi)$  that results in the tightest lower bound is obtained by choosing the smallest possible value of  $C_{N,L}(\phi, \psi)$  satisfying (15), i.e.,

$$C_{N,L}(\phi, \psi) \triangleq \sup_{\zeta} C_N(\zeta, \phi, \psi). \quad (17)$$

Similarly, the optimal  $C_{N,U}(\phi, \psi)$  that results in the tightest upper bound is obtained by

$$C_{N,U}(\phi, \psi) \triangleq \inf_{\zeta} C_N(\zeta, \phi, \psi). \quad (18)$$

Since  $C_N(\zeta, \phi, \psi)$  is monotonically increasing by Theorem 1, we have<sup>4</sup>

$$C_{N,L}(\phi, \psi) = \lim_{\zeta \rightarrow +\infty} C_N(\zeta, \phi, \psi), \quad (19)$$

$$C_{N,U}(\phi, \psi) = \lim_{\zeta \rightarrow 0} C_N(\zeta, \phi, \psi). \quad (20)$$

Without loss of generality, these two limits can be evaluated by assuming  $\zeta_n = \zeta \forall n$  and then taking the limit. Starting from (7) with  $\zeta_n = \zeta \forall n$ , it is easy to see that

$$\lim_{\zeta \rightarrow +\infty} C_N(\zeta, \phi, \psi) = \frac{\int_0^\phi \sin^{2N+2}(\theta + \psi) d\theta}{\int_0^\phi \sin^{2N}(\theta + \psi) d\theta}, \quad (21)$$

$$\lim_{\zeta \rightarrow 0} C_N(\zeta, \phi, \psi) = \left[ \frac{1}{\phi} \int_0^\phi \sin^{2N}(\theta + \psi) d\theta \right]^{1/N}. \quad (22)$$

The above two equations represent the optimum  $C_{N,L}(\phi, \psi)$  and  $C_{N,U}(\phi, \psi)$ , giving (14) as stated.  $\square$

<sup>4</sup>The notations  $\zeta \rightarrow +\infty$  and  $\zeta \rightarrow 0$  are used, respectively, to denote  $\zeta_n \rightarrow +\infty$  and  $\zeta_n \rightarrow 0$  for all  $n$ .

TABLE I  
VALUES OF INTEREST FOR  $S_N(\phi, \psi)$ ,  $C_{N,L}(\phi, \psi)$ , AND  $C_{N,U}(\phi, \psi)$ .

$(\phi, N)$	$S_N(\phi, 0)$	$S_N(\phi, \pi/4)$	$C_{N,L}(\phi, 0)$	$C_{N,U}(\phi, 0)$	$C_{N,L}(\phi, \pi/4)$	$C_{N,U}(\phi, \pi/4)$
$(\Phi_2, 1)$	0.13	0.20	0.75	0.50	0.85	0.82
$(\Phi_2, 2)$	0.10	0.17	0.83	0.61	0.87	0.83
$(\Phi_2, 4)$	0.068	0.13	0.90	0.72	0.91	0.86
$(\Phi_2, 8)$	0.049	0.098	0.94	0.82	0.95	0.89
$(\Phi_4, 1)$	0.23	0.23	0.79	0.61	0.79	0.61
$(\Phi_4, 2)$	0.18	0.18	0.85	0.69	0.85	0.69
$(\Phi_4, 4)$	0.14	0.14	0.90	0.78	0.90	0.78
$(\Phi_4, 8)$	0.098	0.098	0.94	0.85	0.94	0.85
$(\Phi_8, 1)$	0.25	0.23	0.76	0.56	0.78	0.53
$(\Phi_8, 2)$	0.19	0.18	0.83	0.65	0.85	0.64
$(\Phi_8, 4)$	0.14	0.14	0.90	0.75	0.90	0.75
$(\Phi_8, 8)$	0.098	0.098	0.94	0.83	0.94	0.83

Note that, the optimal  $C_{N,L}(\phi, \psi)$  and  $C_{N,U}(\phi, \psi)$  do not depend on the particular  $\zeta$ . In Table I we report some values of interest for  $S_N(\phi, \psi)$ ,  $C_{N,L}(\phi, \psi)$ , and  $C_{N,U}(\phi, \psi)$  with  $\Phi_M \triangleq \pi(M-1)/M$ . For binary PSK signals,  $C_{N,L}(\phi, \psi)$  and  $C_{N,U}(\phi, \psi)$  reduce to

$$C_{N,L}(\Phi_2, 0) = \frac{2N+1}{2N+2}, \quad C_{N,U}(\Phi_2, 0) = \frac{1}{4} \left[ \binom{2N}{N} \right]^{1/N}.$$

Another characteristic of the proposed bounds is that they are asymptotically tight for large  $N$ ; in fact, both  $C_{N,U}(\phi, \psi)$  and  $C_{N,L}(\phi, \psi)$  tend to one, therefore the upper- and lower-bound tend toward each other and hence to the exact solution.

*Remark:* An important result for this class of bounds is that the optimal  $C_{N,L}(\phi, \psi)$  and  $C_{N,U}(\phi, \psi)$  for MRC of INID branches and SSD with IID branches are the same as those for MRC of IID branches.

### III. APPLICATIONS OF THE OPTIMIZED SIMPLE BOUNDS

In this section we apply the optimized simple bounds, developed in the previous section, to the evaluation of the SEP, the SEO, and the SNR penalty.

#### A. Symbol Error Probability

We consider a diversity system with  $N$  available diversity branches<sup>5</sup> employing an arbitrary two-dimensional  $M$ -ary signaling constellation with polygonal decision boundaries. We will examine the case of Rayleigh distributed small-scale fading where the instantaneous symbol SNR on the  $n^{\text{th}}$  diversity branch is exponentially distributed with mean  $\Gamma_n = (a_n/A)\Gamma$ . Here the  $a_n$ 's are related to the power profile of the diversity branches and  $A$  is a normalization factor such that  $\Gamma$  represents the average symbol SNR over all branches ( $A = (1/N) \sum_{n=1}^N a_n$ ).

In [28], the case of SSD with non-ideal channel estimation was considered for systems using arbitrary two-dimensional signaling constellations and operating in IID Rayleigh fading channels. For such systems, it was shown that the SEP as a

<sup>5</sup>In the following we will use the terms path and branch interchangeably since our analysis applies to spatial diversity (e.g., antenna subset diversity), as well as temporal diversity (e.g., selective Rake reception).

function of the SNR,  $\Gamma$ , averaged over the constellation points and the small-scale fading is given by

$$P_e(\Gamma) = \sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} I_N(\zeta^{(i,j)}, \phi_{i,j}, \psi_{i,j}), \quad (23)$$

where  $p_i$  is the transmission probability for constellation point  $s_i$ ,  $\mathcal{B}_i$  is the set consisting of the indices for the constellation points that share a decision boundary with  $s_i$ , and  $\phi_{i,j}$  and  $\psi_{i,j}$  are angles describing the decision region of the  $i^{\text{th}}$  constellation point. The  $n^{\text{th}}$  element of the vector  $\zeta^{(i,j)}$  is given by

$$\zeta_n^{(i,j)} = \frac{b_n}{4} \frac{w_{i,j} N_p \varepsilon \Gamma}{\frac{1}{\Gamma} + N_p \varepsilon + \xi_i}, \quad (24)$$

where the energy corresponding to constellation point  $s_i$  normalized by the mean symbol energy  $E_s$  is given by  $\xi_i = E_i/E_s$ ,  $N_p$  represents the number of received pilot symbols each with energy  $\varepsilon E_s$ ,  $w_{i,j}$  depends on the modulation format, and the set  $\{b_n\}$  is determined by the diversity combining method.

The expression in (23) is valid for subset diversity with IID branches and an arbitrary two-dimensional signaling constellation. Following a derivation like that in [28], it can be shown that the SEP of MRC of INID diversity branches is lower and upper bounded when the following SNR mappings are used:

$$\zeta_{n,L}^{(i,j)} = \frac{a_n}{4A} \frac{w_{i,j} N_p \varepsilon \Gamma}{\frac{A}{a_{\max} \Gamma} + N_p \varepsilon + \xi_i} \quad (25)$$

$$\zeta_{n,U}^{(i,j)} = \frac{a_n}{4A} \frac{w_{i,j} N_p \varepsilon \Gamma}{\frac{A}{a_{\min} \Gamma} + N_p \varepsilon + \xi_i}, \quad (26)$$

where  $a_{\max} = \max_n a_n$  and  $a_{\min} = \min_n a_n$ . As will be apparent from the numerical results presented in Sec. IV, we have found that for SNRs and diversity orders of interest, an SNR mapping which leads to a good approximation is given by

$$\tilde{\zeta}_n^{(i,j)} = \frac{a_n}{4A} \frac{w_{i,j} N_p \varepsilon \Gamma}{\frac{1}{\Gamma} + N_p \varepsilon + \xi_i}. \quad (27)$$

The cases of IID and INID diversity branches can be unified in the form of (23) using appropriate SNR mapping summarized

TABLE II  
SNR MAPPINGS FOR SPECIAL CASES OF THE SEP.

	Subset Diversity		MRC of unequal branch power profile	
	$\zeta_n$ for $M$ -PSK	$\zeta_n^{(*)}$ for $M$ -QAM	$\zeta_n$ for $M$ -PSK	$\zeta_n^{(*)}$ for $M$ -QAM
Ideal Channel Estimation	$b_n c_{\text{MPSK}} \Gamma$	$b_n c_{\text{MQAM}} \Gamma$	$\frac{a_n}{A} c_{\text{MPSK}} \Gamma$	$\frac{a_n}{A} c_{\text{MQAM}} \Gamma$
Non-ideal Channel Estimation	$\frac{b_n N_p \varepsilon c_{\text{MPSK}} \Gamma}{\left(\frac{1}{M} + N_p \varepsilon + 1\right)}$	$\frac{b_n N_p \varepsilon c_{\text{MQAM}} \Gamma}{\left(\frac{1}{M} + N_p \varepsilon + \xi_i\right)}$	$\frac{a_n N_p \varepsilon c_{\text{MPSK}} \Gamma}{A\left(\frac{1}{M} + N_p \varepsilon + 1\right)}$	$\frac{a_n N_p \varepsilon c_{\text{MQAM}} \Gamma}{A\left(\frac{1}{M} + N_p \varepsilon + \xi_i\right)}$

in Table II.<sup>6</sup>

Specifically, for the case of  $M$ -PSK, we have

$$P_e(\Gamma) = 2I_N(\zeta, \Phi_M, 0), \quad (28)$$

and for  $M$ -QAM, we have

$$P_e(\Gamma) = \frac{1}{M} \sum_i \omega_i^{(a)} I_N\left(\zeta^{(i)}, \Phi_2, \frac{\pi}{4}\right) + \frac{1}{M} \sum_i \omega_i^{(b)} I_N\left(\zeta^{(i)}, \Phi_4, 0\right). \quad (29)$$

Note that  $P_e$  depends on  $\Gamma$  through  $\zeta$  as described in Table II with  $c_{\text{MPSK}} = \sin^2(\pi/M)$  and  $c_{\text{MQAM}} = 3/(2(M-1))$ . For each case of interest, these expressions have a compact form and clearly display the dependence of the SEP on the SNR, constellation-size, branch power profile, and diversity technique. The values for  $\omega_i^{(a)}$ ,  $\omega_i^{(b)}$ , and  $\xi_i$  are given in [28, Table I] and the summation in (29) is performed over the nonzero terms.

In the case of SSD with IID branches (i.e., equal branch power profile) we have  $a_n = 1$ ,  $\forall n$  and  $b_n$  can be obtained using the virtual-branch technique [36], [37]. In particular, for H-S/MRC, where the  $L$  strongest branches are combined, the  $b_n$ 's are given by

$$b_n = \begin{cases} 1 & n \leq L \\ L/n & \text{otherwise.} \end{cases}$$

In the case of selection diversity ( $L = 1$ )  $b_n = 1/n$ ,  $\forall n$ , while for the case of MRC  $b_n = 1$ ,  $\forall n$ . For MRC of INID branches,  $b_n = 1$ ,  $\forall n$  and the  $\Gamma_n$ 's are related to the branch power profile through the  $a_n$ 's.

Note that in the case of  $M$ -QAM with ideal channel estimation (29) reduces to

$$P_e(\Gamma) = \frac{\omega^{(a)}}{M} I_N\left(\zeta, \Phi_2, \frac{\pi}{4}\right) + \frac{\omega^{(b)}}{M} I_N(\zeta, \Phi_4, 0), \quad (30)$$

where  $\omega^{(a)} = \sum_i \omega_i^{(a)}$  and  $\omega^{(b)} = \sum_i \omega_i^{(b)}$ , given by  $\{\omega^{(a)}, \omega^{(b)}\} = \{0, 8\}, \{24, 24\}, \{168, 56\}$ , and  $\{840, 120\}$  for  $M = 4, 16, 64$ , and  $256$ , respectively.

Since all terms in (23) are positive, we can directly obtain lower and upper bounds for the SEP of any two-dimensional constellation using Theorem 2 as

$$P_{e,L}(\Gamma) \leq P_e(\Gamma) \leq P_{e,U}(\Gamma), \quad (31)$$

<sup>6</sup>Note that all the SNR mappings given in Table II, and the resulting expressions involving such SNR mappings, are exact for all cases, except those for MRC of unequal branch power profile with non-ideal channel estimation.

$$P_{e,L}(\Gamma) = \sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} I_{N,L}(\zeta^{(i,j)}, \phi_{i,j}, \psi_{i,j}), \quad (32a)$$

$$P_{e,U}(\Gamma) = \sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} I_{N,U}(\zeta^{(i,j)}, \phi_{i,j}, \psi_{i,j}). \quad (32b)$$

Equations (32a) and (32b) can be specialized for  $M$ -PSK as

$$P_{e,L}(\Gamma) = 2I_{N,L}(\zeta, \Phi_M, 0), \quad (33a)$$

$$P_{e,U}(\Gamma) = 2I_{N,U}(\zeta, \Phi_M, 0), \quad (33b)$$

and for  $M$ -QAM as

$$P_{e,L}(\Gamma) = \frac{1}{M} \sum_i \omega_i^{(a)} I_{N,L}\left(\zeta^{(i)}, \Phi_2, \frac{\pi}{4}\right) + \frac{1}{M} \sum_i \omega_i^{(b)} I_{N,L}\left(\zeta^{(i)}, \Phi_4, 0\right), \quad (34a)$$

$$P_{e,U}(\Gamma) = \frac{1}{M} \sum_i \omega_i^{(a)} I_{N,U}\left(\zeta^{(i)}, \Phi_2, \frac{\pi}{4}\right) + \frac{1}{M} \sum_i \omega_i^{(b)} I_{N,U}\left(\zeta^{(i)}, \Phi_4, 0\right). \quad (34b)$$

## B. Symbol Error Outage

In digital mobile radio systems the SEP alone is not sufficient to describe the link quality when a fast process (e.g., thermal noise and small-scale fading) is superimposed on the slow process (e.g., combination of mobility, shadowing, and power control). In such a situation the SEO is a reasonable performance metric since it characterizes the effect of slow variations of the channel on system performance [45]–[48]. For a target SEP equal to  $P_e^*$ , the SEO is defined as:

$$P_o \triangleq \mathbb{P}\{P_e(\Gamma) > P_e^*\}. \quad (35)$$

For mobile radio applications with equal branch power profile, where different paths are affected by the same shadowing level, or applications with unequal branch power profiles, where there is completely correlated shadowing, the vector  $\zeta$  depends on only a single r.v.,  $\Gamma$ , representing the so-called local-mean SNR. Moreover, the function  $P_e(\Gamma)$  is strictly decreasing in its argument and the SEO becomes

$$P_o = \int_0^{\Gamma^*} f_{\Gamma}(y) dy, \quad (36)$$

where  $\Gamma^*$  is the required SNR to achieve the target SEP and  $f_{\Gamma}(\cdot)$  is the probability density function (PDF) of  $\Gamma$ . Hence,

the crucial point in evaluating the SEO is inverting  $P_e$ , that is, finding  $\Gamma^* = P_e^{-1}(P_e^*)$ .

Using (23) and (35) the SEO is given by

$$P_o(P_e^*) = \mathbb{P} \left\{ \sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} I_N(\zeta^{(i,j)}, \phi_{i,j}, \psi_{i,j}) \geq P_e^* \right\}. \quad (37)$$

The analysis of (37) requires an inverse SEP expression which in general requires numerical root evaluation of a function, consisting of a sum of integrals, whose complexity increases with  $N$  and  $M$ . This difficulty can be alleviated by using our optimized bounds that, as will be shown in Sec. IV, are tight for all SNRs, diversity order  $N$ , constellation size  $M$ , and branch power profiles  $\{a_n\}$ , as well as diversity combining methods  $\{b_n\}$ . Moreover, the optimized bounds are easily invertible since they can be written as ratios of polynomials in  $\Gamma$ . The bounds on the SEP in (32) can be used to obtain both upper and lower bounds for  $P_o$  as given in the following. In general, the SEO as a function of target SEP,  $P_e^*$ , is lower and upper bounded by

$$P_{o,L}(P_e^*) \leq P_o(P_e^*) \leq P_{o,U}(P_e^*), \quad (38)$$

where,

$$P_{o,L}(P_e^*) = \int_0^{P_{e,L}^{-1}(P_e^*)} f_\Gamma(y) dy, \quad (39a)$$

$$P_{o,U}(P_e^*) = \int_0^{P_{e,U}^{-1}(P_e^*)} f_\Gamma(y) dy. \quad (39b)$$

In fact, since the SEP decreases monotonically with  $\Gamma$  and  $f_\Gamma(y)$  is non-negative, we obtain (38) and (39) by inverting (32).

At this point some comments can be made regarding the computation of the SNRs,  $\Gamma_L^* = P_{e,L}^{-1}(P_e^*)$  and  $\Gamma_U^* = P_{e,U}^{-1}(P_e^*)$ , required for the derivation of the lower and upper bounds. In particular, since the terms  $I_{N,L}(\cdot)$  and  $I_{N,U}(\cdot)$  of  $P_{e,L}(\cdot)$  and  $P_{e,U}(\cdot)$  in (32) are ratios of polynomials, the equations  $P_{e,L} = P_e^*$  and  $P_{e,U} = P_e^*$  are also polynomials. The exact degree of these polynomials will depend on the signaling constellation, diversity order, and channel estimation method.<sup>7</sup>

1) *SEO for Log-Normal Distributed Shadowing*: It has been shown that shadowing in mobile radio systems is well modeled by the log-normal distribution [54], [55]. In this case,  $\Gamma$  is a log-normal distributed r.v. with parameters  $\mu_{dB}$  and  $\sigma_{dB}$ , that is,  $10 \log_{10} \Gamma$  is a Gaussian r.v. with mean  $\mu_{dB}$  and standard deviation  $\sigma_{dB}$ .<sup>8</sup> Since the logarithm is monotonic, the SEO is lower and upper bounded by:

$$P_{o,L}(P_e^*) = Q \left( \frac{\mu_{dB} - 10 \log_{10} \Gamma_L^*}{\sigma_{dB}} \right), \quad (40a)$$

$$P_{o,U}(P_e^*) = Q \left( \frac{\mu_{dB} - 10 \log_{10} \Gamma_U^*}{\sigma_{dB}} \right), \quad (40b)$$

where  $Q(\cdot)$  is the Gaussian  $Q$ -function (see, e.g., [12]).

<sup>7</sup>For example, for  $M$ -PSK with ideal channel estimation, the polynomials will be of degree  $N$ .

<sup>8</sup>The PDF of  $\Gamma$  is reported in [11].

2) *Inversion of the Optimized Simple Bounds*: From the above discussion it is apparent that the evaluation of the lower- and upper-bound on the SEO requires finding  $\Gamma_L^*$  and  $\Gamma_U^*$ . Note that under some circumstances both  $\Gamma_L^*$  and  $\Gamma_U^*$  can be obtained analytically from known equations for roots of polynomials. For example, the lower and upper bounds on the required SNR  $\Gamma^*$ , for systems employing  $M$ -PSK signaling with MRC of two INID branches in the presence of ideal channel estimation, are given by (41).<sup>9</sup>

$$\Gamma_L^*(P_e^*) = A/(2a_1a_2) \left\{ [C_{2,L}(\phi_M, 0)]^2 (a_1 - a_2)^2 + 4a_1a_2 S_2(\phi_M, 0)/P_e^* \right\}^{1/2} - C_{2,L}(\phi_M, 0)(a_1 + a_2) \quad (41a)$$

$$\Gamma_U^*(P_e^*) = A/(2a_1a_2) \left\{ [C_{2,U}(\phi_M, 0)]^2 (a_1 - a_2)^2 + 4a_1a_2 S_2(\phi_M, 0)/P_e^* \right\}^{1/2} - C_{2,U}(\phi_M, 0)(a_1 + a_2) \quad (41b)$$

However, a numerical root evaluation is needed in general. The polynomial nature of the function implies that the OSBs are easily invertible for all signaling constellations, diversity orders  $N$ , and channel estimation methods, despite the fact that inverting the exact SEP can be time consuming. Since  $I_N(\zeta, \phi, \psi)$  is an integral, inverting the exact SEP requires inversion of a weighted sum of integrals. However, inverting the OSBs requires only the inversion of a weighted sum of ratios of polynomials, which can itself be written as a higher order polynomial. Thus, the OSBs can be inverted more quickly and with less complexity.

### C. SNR Penalty

Starting from the general expression for the SEP of two-dimensional modulation given in (23) it is possible to define the SNR penalty with respect to a reference system. The SNR penalty [27], [28], [38] between a reference system, ‘Ref,’ and the system of interest, ‘B,’ where the reference system outperforms system ‘B,’ is defined as the necessary increase in SNR such that ‘B’ performs as well as ‘Ref’. Thus, the SNR penalty is given by  $\beta(\Gamma)$  such that

$$P_{e,B}(\Gamma) = P_{e,Ref}(\Gamma/\beta(\Gamma)). \quad (42)$$

For large SNR the asymptotic SNR penalty,  $\beta_A$ , is given by

$$P_{Ae,B}(\Gamma) = P_{Ae,Ref}(\Gamma/\beta_A), \quad (43)$$

where  $P_{Ae,B}(\cdot)$  and  $P_{Ae,Ref}(\cdot)$  are the SEP expressions for large SNR (asymptotic behavior).

For a given number of diversity branches  $N$  and constellation, we consider MRC of IID branches with ideal channel estimation as the reference system, since it provides the best performance. We then compare this to the SEP for the cases of 1) MRC of unequal branch power profile (INID branches), and 2) SSD with equal branch power profile (IID branches). In both cases, we consider ideal and non-ideal channel estimation.

<sup>9</sup>The expressions in (41) also hold for SSD of IID branches when  $a_1$  and  $a_2$  are substituted with  $b_1$  and  $b_2$ , respectively, and  $A = 1$ .

The general asymptotic SEP expression for the reference system is given by

$$P_{Ae, \text{Ref}}(\Gamma) = \frac{\sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} S_N(\phi_{i,j}, \psi_{i,j})}{\prod_{n=1}^N \zeta_n} \triangleq \frac{\mathcal{L}_{N,M}}{\Gamma^N}, \quad (44)$$

where  $\mathcal{L}_{N,M}$  depends on both  $N$  and  $M$ . For  $M$ -PSK it is given by

$$\mathcal{L}_{N,M} = \frac{S_N(\Phi_M, 0)}{c_{\text{MPSK}}^N}, \quad (45)$$

and for  $M$ -QAM it results in

$$\mathcal{L}_{N,M} = \frac{\omega^{(a)} S_N(\Phi_2, \frac{\pi}{4}) + \omega^{(b)} S_N(\Phi_4, 0)}{M c_{\text{MQAM}}^N}. \quad (46)$$

The asymptotic SNR penalties for several cases of interest are listed below:

- Maximal ratio diversity for the case of unequal branch power profile with ideal channel estimation:

$$\beta_A = \left( \prod_{n=1}^N \frac{A}{a_n} \right)^{\frac{1}{N}}, \quad (47)$$

- Maximal ratio diversity for the case of unequal branch power profile with non-ideal channel estimation:

$$\beta_A = \left( \prod_{n=1}^N \frac{A}{a_n} \right)^{\frac{1}{N}} \times \left[ \frac{\sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} S_N(\phi_{i,j}, \psi_{i,j}) \left( \frac{N_p \varepsilon + \xi_i}{N_p \varepsilon} \right)^N}{\sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} S_N(\phi_{i,j}, \psi_{i,j})} \right]^{\frac{1}{N}}, \quad (48)$$

- Subset diversity for the case of equal branch power profile with ideal channel estimation:

$$\beta_A = \left( \prod_{n=1}^N \frac{1}{b_n} \right)^{\frac{1}{N}}, \quad (49)$$

- Subset diversity for the case of equal branch power profile with non-ideal channel estimation:

$$\beta_A = \left( \prod_{n=1}^N \frac{1}{b_n} \right)^{\frac{1}{N}} \times \left[ \frac{\sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} S_N(\phi_{i,j}, \psi_{i,j}) \left( \frac{N_p \varepsilon + \xi_i}{N_p \varepsilon} \right)^N}{\sum_{i=1}^M p_i \sum_{j \in \mathcal{B}_i} S_N(\phi_{i,j}, \psi_{i,j})} \right]^{\frac{1}{N}}. \quad (50)$$

*Remark:* Note that the asymptotic SNR penalties for non-ideal channel estimation in (48) and (50) are composed of two terms: the first is the asymptotic SNR penalty related to the branch power profile or diversity combining method, as in (47) and (49), while the second is due to the non-ideal channel estimation which depends on the modulation format and the estimation accuracy.

Wireless communication systems often operate in the moderate or small SNR regime. Therefore it is of interest to obtain the SNR penalty for all values of SNR. The SNR penalty at a given target SEP  $P_e^*$ , such that  $P_e(\Gamma^*) = P_e^*$  for the system

under analysis and  $P_{e, \text{Ref}}(\Gamma_{\text{Ref}}^*) = P_e^*$  for the reference system, is given by

$$\beta_{P_e}(P_e^*) = \frac{P_e^{-1}(P_e^*)}{P_{e, \text{Ref}}^{-1}(P_e^*)} = \frac{\Gamma^*}{\Gamma_{\text{Ref}}^*}. \quad (51)$$

This expression clearly requires inversion of the SEP. As pointed out before, inversion of the exact SEP is, in general, difficult. To make this problem analytically tractable we can use the optimized simple bounds to approximate the SNR penalty. Using the bounds, it can be shown that:

$$\frac{\Gamma_{\text{Ref,L}}^*}{\Gamma_{\text{Ref,U}}^*} \leq \beta_{P_e}(P_e^*) \leq \frac{\Gamma_{\text{U}}^*}{\Gamma_{\text{Ref,L}}^*}. \quad (52)$$

As will be apparent from the numerical results presented in Sec. IV, we have found that an excellent approximation is given by

$$\tilde{\beta}_{P_e}(P_e^*) \approx \frac{\Gamma_{\text{U}}^*}{\Gamma_{\text{Ref,U}}^*}. \quad (53)$$

The quality of this approximation is a direct consequence of the tightness of the OSBs.

As an example, when the reference system uses  $M$ -PSK signaling with MRC of IID branches and ideal channel estimation, the lower and upper OSBs for the required SNR of the reference system are given, respectively, by

$$\Gamma_{\text{Ref,L}}^* = \frac{1}{c_{\text{MPSK}}} \left\{ \left[ \frac{S_N(\Phi_M, 0)}{P_e^*} \right]^{\frac{1}{N}} - C_{N,L}(\Phi_M, 0) \right\} \quad (54a)$$

$$\Gamma_{\text{Ref,U}}^* = \frac{1}{c_{\text{MPSK}}} \left\{ \left[ \frac{S_N(\Phi_M, 0)}{P_e^*} \right]^{\frac{1}{N}} - C_{N,U}(\Phi_M, 0) \right\}. \quad (54b)$$

#### IV. NUMERICAL RESULTS

In general, the design of diversity systems must take into account the joint effect of the number of diversity branches, the combining method, channel estimation, and the channel characteristics. An understanding of the SEP and SEO is necessary to assess the performance of mobile digital communication systems. We will focus on diversity systems where the branches are subject to independent fading, but do not necessarily have the same mean power levels.

In this section we present numerical results for applications of the proposed bounds to the SEP and inverse SEP. We first examine systems employing  $M$ -PSK and  $M$ -QAM signaling with MRC of unequal branch power profiles (i.e., INID branches) in Rayleigh fading and log-normal shadowing. Both ideal and non-ideal channel estimation are investigated for different numbers of diversity branches and branch power profiles. We will consider SEPs in the range  $10^{-3}$  to  $10^{-1}$ , since these are typical values of interest for uncoded systems. As an example, we will focus on channels with exponentially decaying branch power profile parameterized by  $\delta$ . In this case  $a_n = e^{-\delta(n-1)}$  for which

$$\prod_{n=1}^N a_n = e^{-N(N-1)\delta/2} \quad \text{and} \quad A = \frac{e^{-\delta(N-1)}(e^{\delta N} - 1)}{N(e^{\delta} - 1)}.$$

The dependence of the exact SEP on the parameter  $\delta$  is shown in Fig. 1 for systems employing quadrature PSK

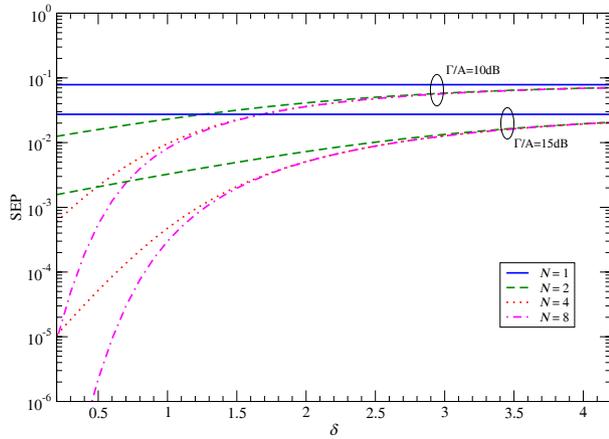


Fig. 1. Exact SEP as a function of  $\delta$  for systems employing QPSK signaling with MRC of INID branches in the presence of ideal channel estimation. Various values of  $\Gamma/A$  and  $N$  are considered.

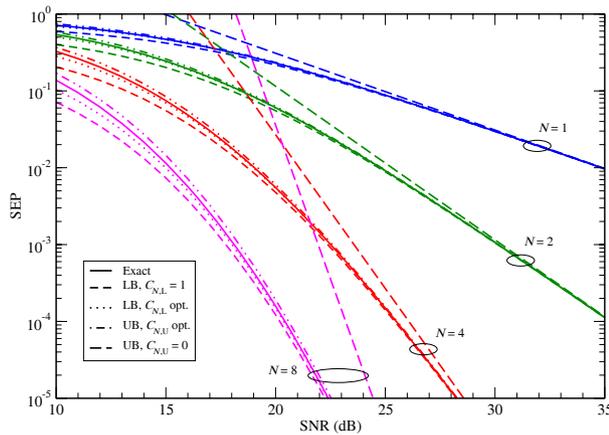


Fig. 2. SEP as a function of  $\Gamma$  (dB) for systems employing 64-QAM signaling with MRC of INID branches ( $\delta = 0.5$ ) in the presence of ideal channel estimation. Exact SEP, lower bound with  $C_{N,L}(\phi, \psi) = 1$ , lower bound with optimum  $C_{N,L}(\phi, \psi)$ , upper bound with optimum  $C_{N,U}(\phi, \psi)$  and upper bound with  $C_{N,U}(\phi, \psi) = 0$  are shown.

(QPSK) signaling with MRC of  $N$  diversity branches,  $\Gamma/A = 10$  dB and 15 dB, in the presence of ideal channel estimation. While  $N$  represents the number of diversity branches, the actual diversity benefit that is achieved depends on  $\delta$  and  $\Gamma/A$ . For example, for  $\Gamma/A = 15$  dB and  $\delta \geq 1.5$  the SEP with 4 branches is almost the same as that with 8 branches. This implies that the two systems capture the same diversity order with only a gain difference in  $\Gamma$  due to the different values of  $A$ . For lower values of  $\Gamma/A$ , the same behavior occurs at a lower value of  $\delta$ . For example, when  $\Gamma/A = 10$  dB the SEPs of the two systems are nearly equal for  $\delta \geq 1.2$ . This figure enables the system designer to quantify the achievable diversity with respect to the available diversity, and to make appropriate choices for system design.

Figs. 2 and 3 show the SEP as a function of the SNR for systems employing 64-QAM signaling with  $N$ -branch MRC in the presence of ideal channel estimation. These figures show the exact SEP; the lower bound given in (4) (i.e.,  $C_{N,L} = 1$ ); our lower and upper OSBs; and the asymptotic upper bound in (2) (i.e.,  $C_{N,U} = 0$ ). Fig. 2 depicts the SEP of INID channels

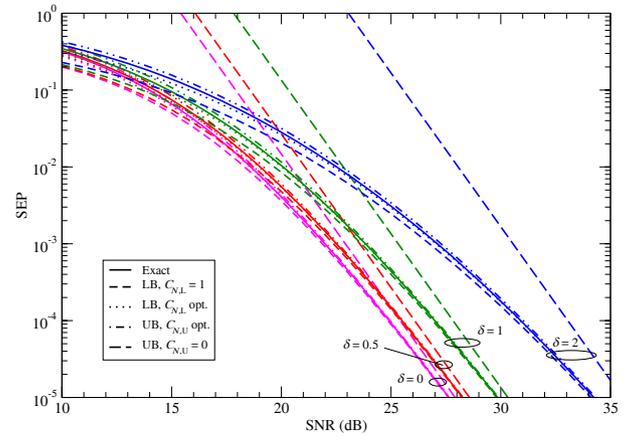


Fig. 3. SEP as a function of  $\Gamma$  (dB) for MRC of IID and INID branches with ideal channel estimation. The case of 64-QAM with  $N = 4$  branches for various values of  $\delta$  is considered. Exact SEP, lower bound with  $C_{N,L} = 1$ , lower bound with optimum  $C_{N,L}$ , upper bound with optimum  $C_{N,U}$  and asymptotic upper bound with  $C_{N,U} = 0$  are shown.

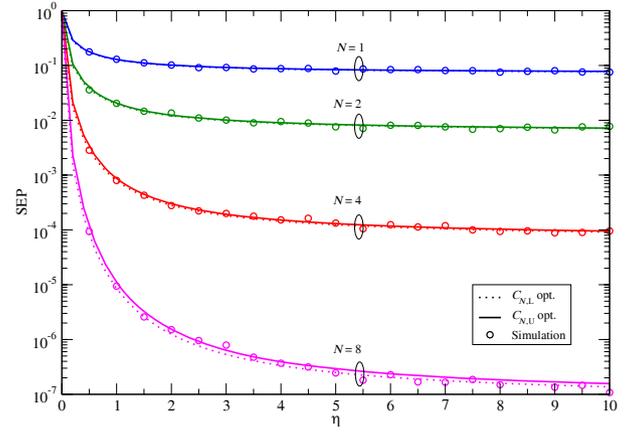


Fig. 4. SEP as a function of  $\eta = N_p \varepsilon$  for systems employing 64-QAM signaling at  $\Gamma = 26$  dB with MRC of INID ( $\delta = 0.5$ ) branches in the presence of non-ideal channel estimation. Lower bound with optimum  $C_{N,L}$  and upper bound with optimum  $C_{N,U}$  are shown together with simulations.

with  $N = 1, 2, 4$ , and 8 diversity branches with  $\delta = 0.5$ . Note that the lower bound in (4) departs from the exact SEP as the number of branches increases. It is remarkable that, unlike the asymptotic upper bound (2) and lower bound (4), the OSBs remain tight for all, including low and moderate, SNRs regardless of the number of branches. Similarly, Fig. 3 shows the SEP with  $N = 4$  branches for IID ( $\delta = 0$ ) and INID ( $\delta = 0.5, 1$ , and 2) channels. Note that for a target SEP of  $10^{-2}$  the asymptotic upper bound is about 1.9 dB away from the exact SEP for IID channels and increases with  $\delta$  (i.e., 2.0, 2.7, and 5.3 dB for  $\delta = 0.5, 1$ , and 2, respectively), whereas the OSBs are only fractions of a dB away from the exact SEP regardless of  $\delta$ .

We now consider the case of non-ideal channel estimation. Specifically, Fig. 4 shows the lower and upper OSBs on the SEP (using the approximate SNR mapping) as a function of  $N_p \varepsilon$  for 64-QAM with  $\delta = 0.5$  and several values of  $N$ . Clearly, the bounds are very close to each other even in the presence of non-ideal channel estimation. The figure also shows the exact symbol error rate obtained through simulations. The simulation results are in agreement with the

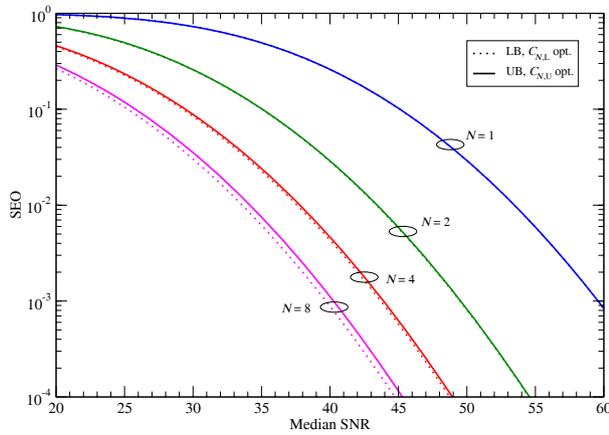


Fig. 5. Upper and lower bounds on the SEO, with optimized  $C_{N,U}$  and  $C_{N,L}$ , as a function of the median SNR,  $\mu_{dB}$ , for systems employing 64-QAM signaling with MRC of INID ( $\delta = 0.5$ ) branches in the presence of ideal channel estimation. Log-normal shadowing with  $\sigma_{dB} = 8$  and a target SEP  $P_e^* = 10^{-2}$  are considered.

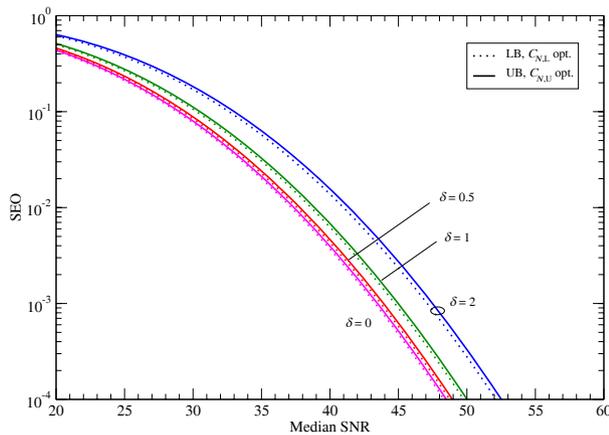


Fig. 6. Upper and lower bounds on the SEO, with optimized  $C_{N,U}$  and  $C_{N,L}$ , as a function of the median SNR,  $\mu_{dB}$ , for systems employing 64-QAM signaling with MRC of IID and INID branches ( $N = 4$ ) in the presence of ideal channel estimation. Log-normal shadowing with  $\sigma_{dB} = 8$  and a target SEP  $P_e^* = 10^{-2}$  are considered.

OSBs, showing that the simple, invertible bounds, based on the approximate SNR mapping, accurately predict the SEP. This figure also shows that, as  $\eta = N_p \varepsilon$  increases, the SEP of a system with non-ideal channel estimation quickly approaches the SEP of an ideal system (i.e.,  $\eta \rightarrow \infty$ ). Thus, even for small values of  $\eta$  we can achieve performance which is close to an ideal system. The ability to quickly produce plots like these, by using the OSBs, allows one to easily determine the required  $N_p \varepsilon$  for a given target SEP, SNR,  $\delta$ , and number of diversity branches.

In addition to the SEP, the SEO is another useful performance measure for the design of digital wireless communication systems. For a fixed SEO corresponding to a given target SEP, the required median SNR,  $\mu_{dB}$ , can be determined. This is useful in the design of digital radio systems with diversity reception, since  $\mu_{dB}$  translates to the maximum distance of a radio-link when the path-loss law is known. In Fig. 5 we show the OSBs on the SEO as a function of the median SNR,  $\mu_{dB}$ , for 64-QAM with ideal channel estimation at a target SEP of  $P_e^* = 10^{-2}$ . We consider  $N = 1, 2, 4$ , and 8 branches

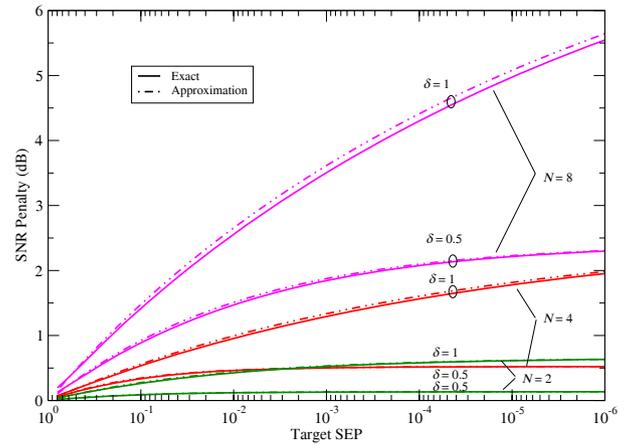


Fig. 7. Exact and approximate SNR penalties in dB as a function of the target SEP,  $P_e^*$ , for systems employing 64-QAM signaling with MRC of INID branches in the presence of ideal channel estimation.

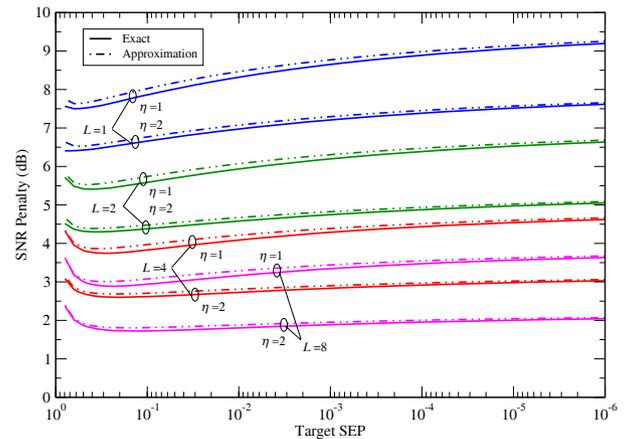


Fig. 8. Exact and approximate SNR penalties in dB as a function of the target SEP,  $P_e^*$ , for systems employing 64-QAM signaling with H-S/MRC of IID branches ( $N = 8$ ) in the presence of non-ideal channel estimation.

and an INID branch power profile with  $\delta = 0.5$  in a log-normal shadowing environment with  $\sigma_{dB} = 8$ . Note that in all cases, the lower and upper bounds are very close to each other, and thus also to the exact solution. Therefore, a system designer can easily use the proposed bounds to choose system parameters without compromising accuracy.

The effect of the parameter  $\delta$  on the SEO is shown in Fig. 6 where the bounds on the SEO as a function of the median SNR,  $\mu_{dB}$ , are plotted for 64-QAM with ideal channel estimation. We consider  $N = 4$  branches and different branch power profiles spanning from IID ( $\delta = 0$ ) to INID ( $\delta = 0.5, 1$ , and 2) channels. The bounds are evaluated for a target SEP equal to  $P_e^* = 10^{-2}$  in log-normal shadowing with  $\sigma_{dB} = 8$ . In all cases the lower and upper bounds are very close to each other and thus to the exact SEO.

In Figs. 7 and 8 we compare the exact and approximate SNR penalty as a function of the target SEP for 64-QAM. For these results a system that employs MRC of IID branches with ideal channel estimation is used as the reference system. The approximations are obtained using (53). First, in Fig. 7 we consider MRC of INID branches ( $\delta = 0.5$  and 1) with

ideal channel estimation and  $N = 2, 4,$  and  $8$  branches. It can be seen that the resulting approximation is very close to the exact penalty in all conditions. Next, in Fig. 8 we consider H-S/MRC of IID branches ( $N = 8$ ) with non-ideal channel estimation characterized by different values of  $N_p \varepsilon$ . At a target SEP of  $10^{-2}$  the difference in SNR penalty between selection diversity ( $L = 1$ ) and MRC ( $L = N = 8$ ) is about 5 dB for  $\eta = N_p \varepsilon = 2$ . In general, the computation of the exact SNR penalty is difficult, whereas it is much easier to use the proposed bounds to closely approximate the SNR penalty. Using this figure one can assess the SNR penalty at a particular target SEP for a specified  $L, N,$  and  $N_p \varepsilon$ . This allows the system designer to make decisions about how many of the available branches to combine and how much energy to devote to the channel estimation process to achieve the desired level of performance.

## V. CONCLUSION

We proposed a new class of optimized bounds for the SEP of systems utilizing arbitrary two-dimensional signaling constellations. Specifically, we consider maximal ratio diversity of INID branches and SSD of IID branches, with both ideal and non-ideal channel estimation. Both the lower and upper bounds are tight (i.e., fractions of a dB from the exact SEP) for all, including low and moderate, SNRs of interest, branch power profiles, diversity techniques, constellation sizes, and amount of energy devoted to channel estimation. This property, together with the fact that they are easily invertible, permits the derivation of tight lower and upper bounds for the inverse SEP. This enables the system designer to obtain several important metrics for mobile radio systems, such as the SEO (i.e., the SEP-based error outage probability) and the SNR penalty for a target SEP in environments with small-scale fading superimposed on shadowing. As an example application, we investigate the performance of  $M$ -PSK and  $M$ -QAM systems in Rayleigh fading and log-normal shadowing environments. The proposed bounds are useful for the design of digital mobile radio systems employing practical diversity techniques.

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