Combined Partial Equalization for MC-CDMA Wireless Systems

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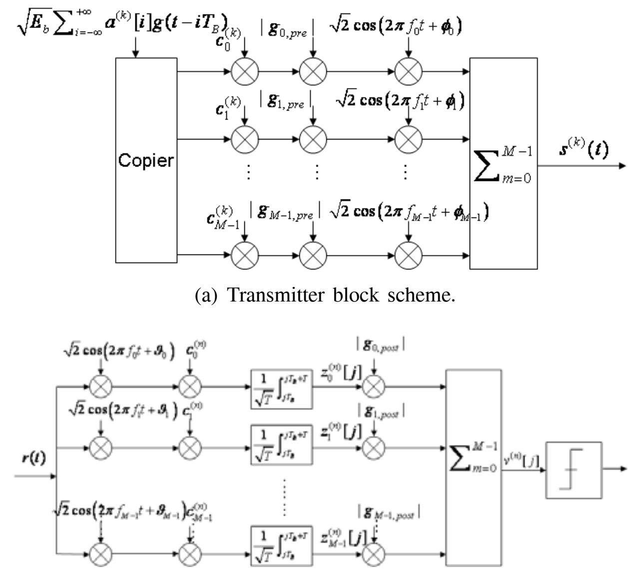
***Abstract*—We analyze a combined equalization technique for multi carrier-code division multiple access (MC-CDMA) systems which consists in performing both pre-equalization at the transmitter and post-equalization at the receiver. In particular, a parametric partial equalization (PE) technique is considered at both sides, and we derive a generalized analytical framework to evaluate the bit error probability (BEP) and choose the optimal PE parameters which minimizes the BEP depending on system parameters.**

***Index Terms*—MC-CDMA, combined equalization, performance evaluation, fading channels.**

I. INTRODUCTION

MULTI carrier-code division multiple access (MC-CDMA) [1]–[3] represents an efficient scheme for robust and high data rate transmission in fading channels with interference. Several techniques to combine signals on each subcarrier are known in the literature, trying to exploit diversity and minimize the effect of fading and multi-user (MAI) interference. In conventional MC-CDMA systems, the MAI mitigation is accomplished at the receiver or at the transmitter using single-user or multiuser detection schemes. In this work we focus on the downlink and we consider linear combining techniques. Different linear schemes based on channel state information (CSI) are known in the literature: maximal ratio combining (MRC), equal gain combining (EGC), orthogonality restoring combining (ORC), and partial equalization (PE) are only some of the most known (see, e.g., [4]).[1] We assume CSI simultaneously available at both the transmitter and the receiver to evaluate if a combined equalization at both sides can improve the system performance in terms of bit error probability (BEP). In particular, we analytically derive the BEP for the downlink of a MC-CDMA system when PE is adopted at both the transmitter and the receiver. PE is a parametric technique which includes previously cited linear techniques and allows the derivation of a general framework to assess the performance evaluation and sensitivity to system parameters. A similar approach was proposed in [5], where the performance was analytically derived in the downlink for a single user case, and in [6], where PE was considered at the transmitter and threshold ORC (TORC) at the receiver. The novelty of this work consists in: (i) considering a multiuser scenario; (ii) analytically evaluating optimal values for PE parameters; (iii) investigating when combined equalization introduces some benefits with respect to classical single side equalization techniques. Simulation results will also be given to confirm the validity of the analysis.



(a) Transmitter block scheme.

(b) Receiver block scheme.

Fig. 1. Transmitter and receiver block schemes.

II. SYSTEM MODEL

We consider the MC-CDMA architecture presented in Fig. 1 where the spreading is performed in the frequency-domain and the number of subcarriers M is equal to the spreading factor. We consider Walsh-Hadamard (W-H) orthogonal codes for the multiple access and binary phase shift keying (BPSK) modulation. The transmitted signal is then given by

$$s(t) = \sqrt{\frac{2E_\mathrm{b}}{M}} \sum_{k=0}^{N_\mathrm{u}-1} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} c_m^{(k)} a^{(k)}[i] \times |g_{\mathrm{pre},m}| g(t-iT_\mathrm{b}) \cos(2\pi f_m t + \phi_m)\,, \qquad (1)$$

where E_b is the energy per bit, N_u is the number of active users, i denotes the data index, m is the subcarrier index, c_m is the $m^{\underline{\mathrm{th}}}$ chip, $a^{(k)}[i]$ is the data-symbol at time i referred to user k, $g_{\mathrm{pre},m}$ is the $m^{\underline{\mathrm{th}}}$ pre-PE coefficient, $g(t)$ is the unitary energy pulse waveform with duration $[0,T]$, T_b is the bit-time, f_m is the $m^{\underline{\mathrm{th}}}$ subcarrier frequency, ϕ_m is the random phase uniformly distributed within $[-\pi,\pi]$, and $T_\mathrm{b} = T + T_\mathrm{g}$ is the total OFDM symbol duration with a time-guard T_g.

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[1] The optimum choice for linear equalization is the minimum mean square error (MMSE) technique, which is more complex since requires not only the CSI, but also the knowledge of signal power, noise power, and number of users.

We normalize the pre-PE coefficients to have the same transmit power as without pre-PE

$$g_{\text{pre},m} = g_m \sqrt{\frac{M}{\sum_{i=0}^{M-1} |g_i|^2}}\,, \tag{2}$$

where g_m is the m^{th} pre-PE coefficient without power constraint given by

$$g_m = h_m^{\star}/|h_m|^{1+\beta_{\text{T}}}\,, \tag{3}$$

being h_m the m^{th} channel gain and β_{T} the PE parameter at the transmitter. Note that when $\beta_{\text{T}} = -1$, 0, and 1, g_m in (3) reduces to the case of MRC, EGC, and ORC, respectively. We consider the downlink, with perfect phase compensation, hence the argument of $g_{\text{pre},m}$ can be included inside ϕ_m in (1), explicitly considering only its absolute value.

As for the channel model, the subcarriers experience channel gains $h_m = \alpha_m e^{j\psi_m}$ assumed independent identically distributed (i.i.d.) complex zero-mean Gaussian random variable (RV) with variance σ_{h}^2, such that $\mathbb{E}\left\{\alpha^2\right\} = 2\sigma_{\text{h}}^2$, where α_m and ψ_m are the m^{th} amplitude and phase coefficients, respectively. The received signal results

$$\begin{aligned} r(t) = &\sqrt{\frac{2E_{\text{b}}}{M}} \sum_{k=0}^{N_{\text{u}}-1} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} \alpha_m c_m^{(k)} a^{(k)}[i] g'(t - iT_{\text{b}}) \\ &\times |g_{\text{pre},m}| \cos(2\pi f_m t + \vartheta_m) + n(t)\,, \end{aligned} \tag{4}$$

where $g'(t)$ is the response to $g(t)$ assumed with unitary energy and duration $T' \triangleq T + T_{\text{d}}$, being $T_{\text{d}} \leq T_{\text{g}}$ the enlargement due to the overall distortion of the channel, $n(t)$ the additive white Gaussian noise (AWGN) with two-side power spectral density $N_0/2$ and $\vartheta_m \triangleq \phi_m + \psi_m$. Focusing, without loss of generality, to the l^{th} subcarrier of the n^{th} user, the receiver performs the correlation at the j^{th} instant (perfect synchronization and phase tracking are assumed) of the received signal with $c_l^{(n)}\sqrt{2}\cos(2\pi f_l t + \vartheta_l)$ (see Fig. 1(b) for details). Then, the decision variable $v^{(n)}$ is obtained by linearly combining the weighted signals from each subcarrier through the l^{th} post-PE coefficient $g_{\text{post},l}$, which is given by

$$g_{\text{post},l} = \frac{(g_{\text{pre},l} h_l)^*}{|g_{\text{pre},l} h_l|^{1+\beta_{\text{R}}}}\,, \tag{5}$$

where β_{R} is the post-PE parameter. After some mathematical manipulation, we obtain the decision variable[2]

$$\begin{aligned} v^{(n)} = &\overbrace{\sqrt{\frac{E_{\text{b}}\delta_d}{M}} \sum_{l=0}^{M-1} \alpha_l^{(1-\beta_{\text{T}})(1-\beta_{\text{R}})} a^{(n)}}^{U} \\ &+ \overbrace{\sqrt{\frac{E_{\text{b}}\delta_d}{M}} \sum_{l=0}^{M-1} \sum_{k=0,k\neq n}^{N_u-1} \alpha_l^{(1-\beta_{\text{T}})(1-\beta_{\text{R}})} c_l^{(n)} c_l^{(k)} a^{(k)}}^{I} \\ &+ \overbrace{\sum_{l=0}^{M-1} \alpha_l^{-\beta_{\text{R}}(1-\beta_{\text{T}})} n_l \sqrt{\frac{\sum_{i=0}^{M-1} \alpha_i^{-2\beta_{\text{T}}}}{M}}}^{N}\,, \end{aligned} \tag{6}$$

where $\delta_{\text{d}} = 1/(1 + T_{\text{d}}/T)$ and U, I, and N represent the useful, interference, and noise term, respectively.

[2]For the sake of conciseness in our notation, since ISI is avoided, we will neglect the time-index j in the following.

III. Performance Evaluation

The BEP expression can be obtained by deriving the statistic distributions of the decision variable and, thus, those of U, N, and I in (6). In particular, it is reasonable, for sufficiently high number of subcarriers (as for practical systems, such as digital video broadcasting and WiMAX), to adopt the law of large number (LLN) and the central limit theorem (CLT). By applying the CLT to the term U in (6), it results Gaussian distributed with mean value given by

$$\begin{aligned} \mu_U &= \sqrt{E_{\text{b}}\delta_{\text{d}}M}\ \mathbb{E}\left\{\alpha_l^{(1-\beta_{\text{T}})(1-\beta_{\text{R}})}\right\} \\ &= \sqrt{E_{\text{b}}\delta_{\text{d}}M}(2\sigma_{\text{h}}^2)^{\frac{1}{2}(\beta_{\text{T}}-1)(\beta_{\text{R}}-1)}\Gamma\left[\frac{3+\beta_{\text{T}}(\beta_{\text{R}}-1)-\beta_{\text{R}}}{2}\right] \end{aligned}$$

where $\Gamma[\cdot]$ is the Euler Gamma function. For the interference term, by exploiting the properties of orthogonal codes and applying the CLT as in [4], I results distributed as a zero-mean Gaussian RV with variance

$$\begin{aligned} \sigma_I^2 = &E_{\text{b}}\delta_d(N_{\text{u}}-1)(2\sigma_{\text{h}}^2)^{(\beta_{\text{T}}-1)(\beta_{\text{R}}-1)} \\ &\times \left(\Gamma[2+\beta_{\text{T}}(\beta_{\text{R}}-1)-\beta_{\text{R}}] - \Gamma^2\left[\frac{3+\beta_{\text{T}}(\beta_{\text{R}}-1)-\beta_{\text{R}}}{2}\right]\right)\,. \end{aligned} \tag{7}$$

For the noise term N, be means of the CLT we noticed that

$$\frac{1}{M}\sum_{l=0}^{M-1} \alpha_l^{-2\beta_{\text{T}}} \simeq \frac{1}{M} M\ \mathbb{E}\left\{\alpha^{-2\beta_{\text{T}}}\right\} = (2\sigma_{\text{h}}^2)^{-\beta_{\text{T}}}\Gamma[1-\beta_{\text{T}}] \tag{8}$$

thus N is a zero-mean Gaussian RV with variance

$$\sigma_N^2 = M\frac{N_0}{2}(2\sigma_{\text{h}}^2)^{-\beta_{\text{T}}}\Gamma[1-\beta_{\text{T}}]\mathbb{E}\left\{\alpha^{-2\beta_{\text{R}}(1-\beta_{\text{T}})}\right\}\,, \tag{9}$$

$$\mathbb{E}\left\{\alpha^{-2\beta_{\text{R}}(1-\beta_{\text{T}})}\right\} = (2\sigma_{\text{h}}^2)^{-\beta_{\text{R}}(1-\beta_{\text{T}})}\Gamma[1+\beta_{\text{R}}(\beta_{\text{T}}-1)]\,. \tag{10}$$

Since $a^{(k)}$ is zero mean and statistically independent of α_l and n_l, and considering that n_l and α_l are statistically independent and zero mean, then $\mathbb{E}\left\{IN\right\} = \mathbb{E}\left\{IU\right\} = 0$. Since n_l and α_l are statistically independent, then $\mathbb{E}\left\{NU\right\} = 0$. Moreover I, N, and U are uncorrelated Gaussian RVs, thus also statistically independent. By applying the LLN to the useful term, that is by approximating U with its mean value, the BEP averaged over small-scale fading results

$$P_{\text{b}} \simeq \frac{1}{2}\text{erfc}\sqrt{\Xi}\,, \tag{11}$$

where Ξ is the signal-to-noise plus interference-ratio (SNIR) given by (12) (next page) and $\overline{\gamma} \triangleq E_{\text{b}}\delta_{\text{d}}2\sigma_{\text{h}}^2/N_0$ is the mean signal-to-noise-ratio (SNR) averaged over small-scale fading.

We aim at deriving the optimal choice of the PE parameters, thus the couple (β_{T}, β_{R}) jointly minimizing the BEP

$$(\beta_{\text{T}}, \beta_{\text{R}})^{(\text{opt})} = \arg\min_{\beta_{\text{T}},\beta_{\text{R}}} \left\{P_{\text{b}}(\beta_{\text{T}}, \beta_{\text{R}}, \overline{\gamma})\right\}\,. \tag{13}$$

However, being in the downlink, the receiver is in the mobile unit, hence it is typically more convenient, if necessary, to optimize the parameter at the transmitter (i.e., at the base station), once fixed at the receiver. Therefore, we find the optimum values of β_{T} defined as that values within the range $[-1, 1]$ that minimizes the BEP for each β_{R}

$$\beta_{\text{T}}^{(\text{opt})} = \arg\min_{\beta_{\text{T}}} \left\{P_{\text{b}}(\beta_{\text{T}}, \beta_{\text{R}}, \overline{\gamma})\right\} \simeq \arg\max_{\beta_{\text{T}}} \left\{\Xi\right\}\,. \tag{14}$$

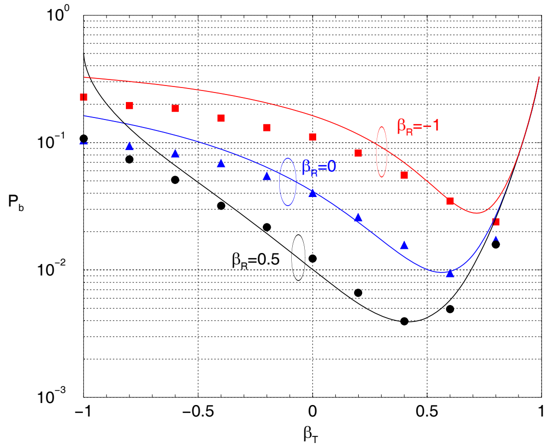

Fig. 2. BEP vs. β_T for different β_R and $\overline{\gamma} = 10$ dB in fully loaded system conditions ($N_u = M = 64$). Comparison among analytical and simulative results are shown.

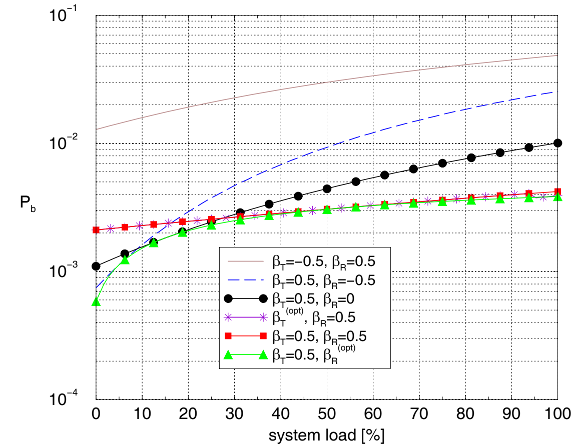

Fig. 3. BEP vs. the system load (N_u/M) for various β_T and β_R when $\overline{\gamma} = 10$ dB.

By deriving (12) with respect to β_T and after some mathematical manipulation, we obtain the implicit solution given by (15), where $\xi \triangleq 2\overline{\gamma}(N_u - 1)/M$ quantifies how much the system is noise-limited (low values) or interference-limited (high values) and $\Psi[\cdot]$ is the logarithmic derivative of the Gamma function.

IV. Numerical Result

In Fig. 2, the BEP is plotted as a function of β_T for different values of β_R and mean SNR $\overline{\gamma} = 10$ dB in fully loaded system conditions ($M = N_u = 64$). Note that, in spite of the post-PE technique, there is always an optimum value of β_T minimizing the BEP and this value depends on β_R. Moreover, the BEP is also drastically dependent on β_R, meaning that a not suitable post-PE technique can even deteriorate the performance, with respect to one side combination, rather than improving it. Simulation results are also reported confirming the analysis especially in correspondence to the optimal β_R (note that the analysis is confirmed for 64 subcarriers and thus it is expected to be even more accurate for higher number of subcarriers).[3] In Fig. 3, the BEP as a function of the system load $(N_u - 1)/M$ in percentage is shown for $\overline{\gamma} = 10$dB and different couples (β_T, β_R). Note how a suitable choice of pre- and post-PE parameters can increase the sustainable system load.

V. Conclusions

In this letter we derived an analytical framework for assessing the performance of downlink MC-CDMA systems with pre- and post-PE. The optimal value of PE parameters is provided showing that it is fundamental for improving the performance in terms of BEP averaged over small-scale fading. Without a proper choice of PE parameters, the performance would be deteriorated with respect to single-side detection. Simulation results confirm the accuracy of the analysis. The gain achieved by a suitable combination of transmission and reception equalization parameters could be exploited to save energy or increase the coverage range.

[3]Similar considerations can be drawn for time- and -frequency correlated SUI-x channels as shown, by simulation, in [7] referred to PE at the receiver.

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$$\Xi \triangleq \frac{\overline{\gamma}\,\Gamma^2\left[\frac{3+\beta_T(\beta_R-1)-\beta_R}{2}\right]}{\Gamma[1-\beta_T]\Gamma[1+\beta_R(\beta_T-1)] + 2\overline{\gamma}\frac{N_u-1}{M}\left(\Gamma[2+\beta_T(\beta_R-1)-\beta_R] - \Gamma^2\left[\frac{3+\beta_T(\beta_R-1)-\beta_R}{2}\right]\right)} \tag{12}$$

$$\xi = \frac{\Gamma[1-\beta_T]\Gamma[1+\beta_R(\beta_T-1)]\left\{-(\beta_R-1)\Psi\left[\frac{3+\beta_T(\beta_R-1)-\beta_R}{2}\right] - \Psi[1-\beta_T] + \beta_R\Psi[1+\beta_R(\beta_T-1)]\right\}}{(\beta_R-1)\Gamma[2+\beta_T(\beta_R-1)-\beta_R]\left\{\Psi\left[\frac{3+\beta_T(\beta_R-1)-\beta_R}{2}\right] - \Psi[2+\beta_T(\beta_R-1)-\beta_R]\right\}} \tag{15}$$