OPTIMUM METRIC FOR FRAME SYNCHRONIZATION WITH GAUSSIAN NOISE AND UNEQUALLY DISTRIBUTED DATA SYMBOLS

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ABSTRACT

The problem of frame synchronization with equiprobable data symbols is widely analyzed in the literature and the optimum metric for AWGN channels has been derived both in the case of periodically and aperiodically embedded data symbols. On the contrary, the case of non-equiprobable data symbols has not been studied, although it can occur in many practical situations.

An optimum metric for frame synchronization in data streams with non-uniformly distributed data symbols is derived in this paper and its performance is investigated through simulation. A performance comparison with the metric which is optimal for equiprobable data symbols is also provided. Results show that the derived optimum metric results in an evident gain, at the expense of a small additional complexity.

Index Terms— Frame synchronization, non-uniform distribution, hypothesis testing, detection.

I. INTRODUCTION

Frame synchronization is one of the key issues in digital communication systems [1]–[4]. The most widely used method for providing frame synchronization is to insert a fixed symbol pattern or sync word into the data stream. Assuming that symbol synchronization has already been obtained, the receiver obtains frame synchronization by locating the position of the sync word in the received data stream. In some applications frame synchronization (FS) can be very critical. Important examples include the transmission of data through wireless links where, due e.g. to the use of powerful error correcting codes, the receiver is designed to work with very low signal to noise ratios.

A first, intuitive approach to FS consists of correlating, within a window where we are sure there is one and only one inserted sync word (SW), the received signal with the expected SW, looking for the position where this correlation is maximum. This can be done only for the periodically embedded case, i.e. with sync words equally spaced (or, in other words, for constant, known, frame lengths).

In the binary symmetric channel (i.e. hard decisions provided to the synchronizer) this detection through correlation is optimal, whereas in the additive white Gaussian noise (AWGN) channel this is not true, as demonstrated in [5]. Regardless of its sub-optimality, detection through correlation has become a common engineering practice.

The problem of frame synchronization has been widely studied in the literature in the case of equiprobable data symbols: the performance evaluation for frame synchronization with periodically embedded sync words searched through correlation in binary symmetric channels (BSC) has been studied in [1], where also synchronization sequences with good aperiodic autocorrelation properties have been identified; in [3] a basic theory of frame synchronization is presented and considerations on the marker design are made. The problem of optimum frame synchronization has been afforded in [5] on AWGN; the optimal metric for AWGN channel has been identified for the considered case of fixed length frames and equiprobable data symbols. In [6] a performance evaluation through simulation of these metrics has been presented. In [7] an extension of Massey's work to derive the optimum maximum likelihood decision rule for general M-ary phase-coherent and phase-noncoherent signaling over the additive white Gaussian noise channel is provided. A union lower bound on synchronization probability for the correlation rule on AWGN channels is also determined. In the same paper, the performance evaluation of the optimal metric [5] is studied via simulation, also in the general case of M-ary signaling. Synchronization for unknown frame lengths is studied by the authors in [8]–[10].

In many practical situations the assumption of uniform data distribution is not realistic: it is shown in [11] that non-equiprobable signaling is of interest in Gaussian channels when the aim is to minimize the average transmitted power. Similarly, in [12] variable-rate data transmission schemes, in which constellation points are selected according to a nonuniform probability distribution, are investigated. When frame synchronization is performed at the physical layer, the case of non-equiprobable data symbols is thus of interest. On the other side, also when frame synchronization is performed at the application layer, i.e. in the case of video streams, it may happen that bits are not equiprobable, since current source coding schemes produce a bitstream made of non-necessarily equiprobable data.

In [13] the authors studied the performance of frame synchronization for non equiprobable data symbols with the metrics derived in [9], [10]. In this paper we derive the optimal metric in this case.

The paper is organized as follows: in Section II the problem of frame synchronization is presented and mathematically formulated. The optimum metric is derived in Section III and its performance is evaluated through simulation in Section IV.

II. PROBLEM STATEMENT

We will use the notation in [10], in the following shortly reviewed and specialized to the case under investigation.

In the case of binary signalling we consider data symbols $d_i \in \{+1, -1\}$, with probabilities $Pr(d_i = 1) = p_1$, $Pr(d_1 = -1) = 1 - p_1$, SW symbols $c_i \in \{+1, -1\}$ and i.i.d. noise samples n_i. The demodulator output consists of a sequence of N symbols, the sampled matched filter outputs. Let this sequence (a random variable) be denoted by $\mathbf{R}$; the actual value of the received vector is the sequence $\mathbf{r} = (r_1, ..., r_N)$, and is composed of either SW symbols and noise or the sum of data symbols and noise. Transmission is over an additive white Gaussian noise channel (AWGN) whose samples are i.i.d with zero mean and variance $N_0/2$, with N_0 the one-sided power spectral density.

In the case of binary modulation, the $i^{\underline{\text{th}}}$ transmitted bit $b_i \in \{0, 1\}$ gives rise, after binary antipodal modulation, transmission through the AWGN channel, matched filter reception and perfect sampling, to a sample $r_i = (-1)^{b_i} + n_i$, where n_i are independent, identically distributed (i.i.d.) Gaussian random variables (r.v.'s), with zero mean and variance σ^2. This model is also valid for BPSK systems, for which the signal-to-noise ratio is $E_s/N_0 = 1/(2\sigma^2)$, where E_s is the energy per symbol.

We assume that a sync word composed of N symbols, $(c_1, ..., c_N)$, is aperiodically inserted in the data stream, composed of symbols d_i.

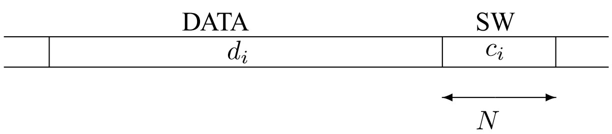

Fig. 1. Frame Structure.

The acquisition algorithm we consider is as follows: starting from a position k, the synchronizer observes a vector of N subsequent samples; based on a suitable metric evaluated from this vector it decides if the SW is in position k; if not, it moves to position $k+1$, repeating the steps until the sync word is detected.

We afford here the problem of deciding at each position k of the bitstream whether a sync word is present or not. The relation between this problem and other performance indicators such as, e.g., the probability of correct acquisition in one pass, is addressed in [3], [14].

As in our previous work, we assume that the statistical properties of the metric do not depend on the position in the bitstream. We thus avoid considering the effect of the aperiodic autocorrelation around SW's. We have evaluated in fact that, if the SW is properly chosen as a sequence with optimized aperiodic autocorrelation property (e.g. Barker sequences [1], or those in [3], [15], [16]) its symbols should mimic random data, with, moreover, the additional property that some configurations can be avoided. We thus neglect in the design of the frame synchronizer the case of "mixed data", i.e., when both data and SW symbols are present in the metric evaluation window. Simulation results are provided in [10] for the case of "mixed data" to support this approximation. In particular we will show that if the SW is properly designed the case of purely random data represents generally a worst case in terms of probability of false sync word detection respect to the "mixed data" case.

As in [10] we study the problem through the statistical theory of hypothesis testing. After observing N subsequent samples, the synchronizer must choose between two possible situations

$$\begin{aligned} \mathcal{H}_0 &: \quad r_i = d_i + n_i, \quad i = 1, \ldots, N \\ \mathcal{H}_1 &: \quad r_i = c_i + n_i, \quad i = 1, \ldots, N \end{aligned} \tag{1}$$

the first hypothesis representing the case where there is no sync word, the second corresponding to the case the sync word is present. Decisions are indicated by $\mathcal{D}_0, \mathcal{D}_1$, corresponding to the "true" hypotheses $\mathcal{H}_0, \mathcal{H}_1$, respectively.

Differently from previous studies, we assume here that the data symbols d_i are not necessarily equiprobable and that their probabilities of occurrence in the data stream are known.

III. DERIVATION OF THE OPTIMAL METRIC

Assuming AWGN and soft values available, the likelihood ratio test (LRT) [17] is considered for the derivation of the optimal detection metric.

In the following we use capital letters to indicate random variables and bold for vectors.

By indicating with $\mathbf{R} = (R_1, ..., R_N)$ the r.v. corresponding to the vector $\mathbf{r} = (r_1, ..., r_N)$ of received samples, the LRT is

$$\Lambda'(\mathbf{r}) = \frac{f_{\mathbf{R}|\mathcal{H}_0}(\mathbf{r}|\mathcal{H}_0)}{f_{\mathbf{R}|\mathcal{H}_1}(\mathbf{r}|\mathcal{H}_1)} \underset{\mathcal{D}_1}{\overset{\mathcal{D}_0}{\gtrless}} \lambda' \tag{2}$$

where $f_{\mathbf{R}|\mathcal{H}_j}(\mathbf{r}|\mathcal{H}_j)$ is the probability density function (p.d.f.) of $\mathbf{R}$ under hypothesis $\mathcal{H}_j$, $j = 0, 1$, and λ' is the selected threshold. Thus, according to the test, $\Lambda'(\mathbf{r}) < \lambda'$ corresponds to the decision $\mathcal{D}_1$, i.e. we decide we are in presence of a sync word; otherwise, the decision is $\mathcal{D}_0$. Since the channel is memoryless, we have also

$$f_{\mathbf{R}|\mathcal{H}_j}(\mathbf{r}|\mathcal{H}_j) = \prod_{i=1}^{N} f_{R_i|\mathcal{H}_j}(r_i|\mathcal{H}_j). \tag{3}$$

We now consider the first equation in (1) with data symbols $d_i \in \{+1, -1\}$. The LRT is then obtained following the method in [10], by observing that $R_i|\mathcal{H}_1$ is Gaussian with mean value c_i and $R_i|(\mathcal{H}_1, d_i)$ are Gaussian r.v.s with variance σ^2 and mean d_i.

We can substitute the numerator in (2) with:

$$f_{R_i|\mathcal{H}_0}(r_i|\mathcal{H}_0) = \tag{4}$$
$$\frac{1}{\sqrt{2\pi}\sigma}\left[p_1 e^{-\frac{(r_i-1)^2}{2\sigma^2}} + (1-p_1)e^{-\frac{(r_i+1)^2}{2\sigma^2}}\right]$$

Since $R_i|H_1$ is complex Gaussian with mean c_i^2 and variance σ^2, we obtain:

$$\Lambda'(\mathbf{r}) = \prod_{i=1}^{N} \frac{p_1 e^{-\frac{(r_i-1)^2}{2\sigma^2}} + (1-p_1)e^{-\frac{(r_i+1)^2}{2\sigma^2}}}{e^{-\frac{|r_i-c_i|^2}{2\sigma^2}}} \tag{5}$$

Since in the binary case we have $|d_i| = |c_i|$:

$$\Lambda'(\mathbf{r}) = \prod_{i=1}^{N}\left[Pr(d_i = c_i) + Pr(d_i \neq c_i)e^{-\frac{2r_i c_i}{\sigma^2}}\right] \underset{\mathcal{D}_1}{\overset{\mathcal{D}_0}{\gtrless}} \lambda'. \tag{6}$$

From this, by applying the logarithm operator, we obtain:

$$\Lambda(\mathbf{r}) = \sum_{i=1}^{N} \log\left[Pr(d_i = c_i) + Pr(d_i \neq c_i)e^{-\frac{2r_i c_i}{\sigma^2}}\right] \underset{\mathcal{D}_1}{\overset{\mathcal{D}_0}{\gtrless}} \lambda. \tag{7}$$

We thus decide $\mathcal{D}_1$ (the start code is present) if $\Lambda''(\mathbf{r}) < \lambda$, $\mathcal{D}_0$ otherwise. The threshold is chosen according to the Neyman-Pearson criterion, i.e. by fixing the maximum tolerable probability of false alarm (emulation). We should note that the obtained metric, similar as the one for the equiprobable case, depends on channel conditions through σ^2 and thus the synchronizer requires the instantaneous knowledge of the signal to noise ratio in order to perform optimum detection. Furthermore, the evaluation of a non-linear function is required.

IV. PERFORMANCE EVALUATION

We report in this section numerical results obtained with the optimum metric derived, also in comparison with results obtained with the metric obtained in the hypothesis of equiprobable data symbols (LRTU).

Results are presented in terms of probability of emulation, P_{EM}, or false start code detection, and of probability of missed detection, P_{MD}.

The probability of emulation, P_{EM}, or false start code detection, of choosing hypothesis $\mathcal{H}_1$ when $\mathcal{H}_0$ is true is

$$P_{EM} = \Pr\{\mathcal{D}_1|\mathcal{H}_0\}. \tag{8}$$

Note that here the false start code detection is due to the case where random data plus noise is interpreted as a SW. This can occur either in the case data symbols are coincident with the SW pattern or, due to noise, even if data symbols are different from the SW pattern.

The probability of missed detection, P_{MD}, of choosing $\mathcal{H}_0$ when $\mathcal{H}_1$ is true, is

$$P_{MD} = \Pr\{\mathcal{D}_0|\mathcal{H}_1\} \tag{9}$$

and the probability of correct detection is

$$P_D = 1 - P_{MD}. \tag{10}$$

By analyzing such probabilities for different values of the threshold λ, we can draw a receiver operating characteristics (ROC) curve, reporting P_D versus P_{EM}.

In the following figures, we report in the same plot different ROC curves, each related to a specific probability distribution of data symbols.

For comparison, the ROC curves obtained for the LRTU metric are reported in Fig. 2, also for different probabilities of data symbols. The considered SW is the Turyn [18] $N = 32$ SW. This SW is composed of 19 symbols equal to 1 and 13 symbols equal to -1. Given the prevalence of symbols equal to one, the cases with $Pr(d_i = 1) < 0.5$ provide better results. AWGN with $E_s/N_0 = -5dB$ is here considered. Such curves have been derived analytically [13] and numerical results have also been confirmed through simulation.

Fig. 3 reports results obtained with the metric derived in this paper, in the same conditions as in Fig. 2. Results are obtained here through simulation.

We can observe that, as expected, for any considered data distribution the metric proposed in this paper outperforms the metric derived under the assumption of equiprobable data symbols. The gain is in particular high for strongly unbalanced data distributions (*e.g.*, $Pr(d_i = 1) = 0.1$ and $Pr(d_i = 1) = 0.9$). We also observe a strong dependency of the performance on the prevalence of symbols equal to $+1$ or -1 in the synchronization word. Note that in the case reported in Figure 3 the worst results are achieved when $Pr(d_i = 1) = 0.6$, which approximates the distribution of 1's in the SW.

V. CONCLUSIONS

The optimal metric for frame synchronization with non-equiprobable data symbols with known distribution is derived in this paper, considering transmission over an AWGN channel.

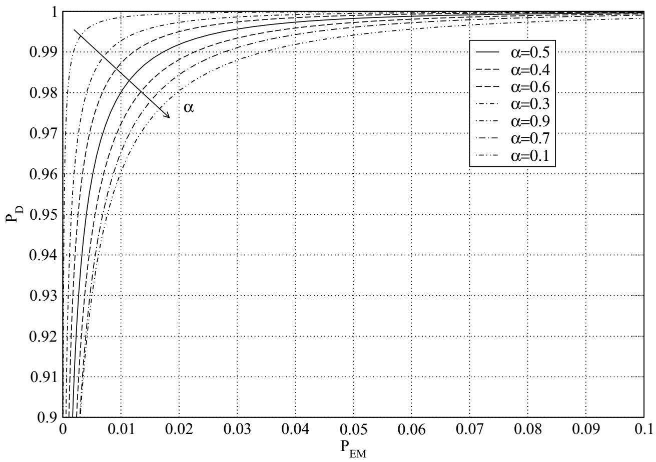

Fig. 2. Receiver Operating Characteristic (ROC) curves. LRTU metric; Turyn [18] sync word of $N = 32$ bits. $\sigma^2 = 2dB; E_s/N_0 = -5dB$.

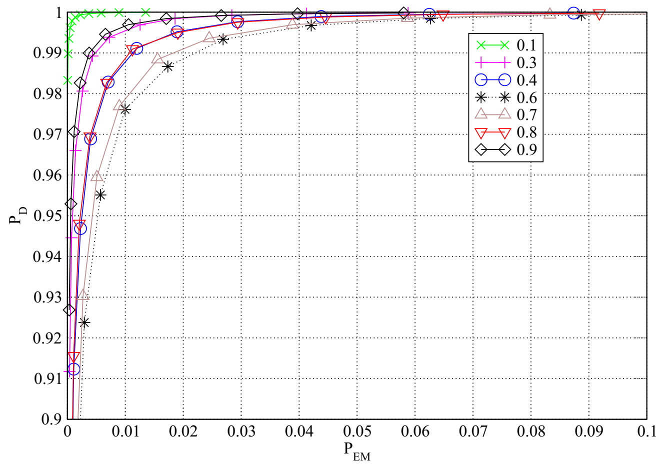

Fig. 3. Receiver Operating Characteristic (ROC) curves. Optimal metric; Turyn [18] sync word of $N = 32$ bits. $\sigma^2 = 2dB; E_s/N_0 = -5dB$.

The performance of the optimal metric, based on likelihood ratio test (LRT), is evaluated via simulation and compared with the performance of the metric obtained with the assumption of equiprobable data symbols (LRTU).

Results show that the gap between the two metrics is evident in particular in the case of very unbalanced data distributions. In the case the distribution of the data symbols is known, the performance of frame synchronization improves remarkably with the use of the proposed metric.

Furthermore, a proper choice of the SW can further improve the performance, since the SWs of practical use have been derived with the assumption of equiprobable data or of absence of a-priori knowledge of the data distribution. In this work we assumed a given synchronization word and we did not focus on its selection.

Future work will consist in an analytical performance evaluation of such a metric.

ACKNOWLEDGMENT

This work was partially supported by the European Commission under FP7 project "OPTIMIX".

VI. REFERENCES

[1] R. H. Barker, "Group synchronization of binary digital systems," in *Communication Theory*. London, England: W. Jackson, 1953.

[2] J. J. Stiffler, *Theory of Synchronous Communications*. Englewood Cliffs, N.J.: Prentice-Hall, 1971.

[3] R. A. Scholtz, "Frame synchronization techniques," *IEEE Trans. on Commun.*, vol. 28, no. 8, pp. 1204–1213, 1980.

[4] C. Georghiades and E. Serpedin, *Communication handbook*. CRC Press, 2002, ch. 18.

[5] J. L. Massey, "Optimum frame synchronization," *IEEE Trans. on Commun.*, vol. 20, no. 4, pp. 115–119, 1972.

[6] P. T. Nielsen, "Some optimum and suboptimum frame synchronizers for binary data in Gaussian noise," *IEEE Trans. on Commun.*, vol. 21, no. 6, pp. 770–772, 1973.

[7] G. L. Lui and H. H. Tan, "Frame synchronization for Gaussian channels," *IEEE Trans. on Commun.*, vol. 35, no. 8, pp. 818–829, 1987.

[8] M. Chiani and M. G. Martini, "Optimum synchronization of frames with unknown, variable lengths on Gaussian channels," in *Proc. IEEE Global Telecomm. Conf.*, Dallas, TX, Nov. 2004, pp. 4087–4091.

[9] ——, "Practical frame synchronization for data with unknown distribution on AWGN channels," *IEEE Commun. Lett.*, vol. 9, no. 5, pp. 456–458, May 2005.

[10] ——, "On sequential frame synchronization in AWGN channels," *IEEE Trans. Commun.*, vol. 54, no. 2, February 2006.

[11] A. R. Calderbank and L. H. Ozarow, "Nonequiprobable signaling on the Gaussian channel," *IEEE Trans. on Information Theory*, vol. 36, no. 4, pp. 726–740, July 1990.

[12] F. R. Kschischang and S. Pasupathy, "Optimal nonuniform signaling for Gaussian channels," *IEEE Trans. on Information Theory*, vol. 39, no. 3, pp. 913–929, May 1993.

[13] M. Chiani and M. G. Martini, "Frame synchronization for unequally distributed data symbols," in *Proc. of IEEE Globecom 2006*, San Francisco, Nov-Dec. 2006.

[14] P. T. Nielsen, "On the expected duration of a search for a fixed pattern in random data," *IEEE Trans. on Information Theory*, no. 9, pp. 702–704, September 1973.

[15] J. L. Maury and F. Styles, “Development of optimum frame synchronization codes for Goddard Space Flight Center PCM telemetry standards,” in *Proc. Nat. Telemetring Conf.*, Los Angeles, CA, June 1964, pp. 1–10.

[16] F. Neuman and L. Hofman, “New pulse sequences with desirable correlation properties,” in *proc. Nat. Telemetry conf.*, Washington, D.C., Apr. 1971, pp. 272–282.

[17] H. L. Van Trees, *Detection, Estimation and Modulation Theory: Part I.* New York: Wiley, 1968.

[18] R. Turyn, “Sequences with small correlation,” in *Error Correcting Codes*, H. B. Mann, Ed. New York: Wiley, 1968.