Adaptive Modulation Systems Subject to Interference

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***Abstract*—Adaptive modulation techniques enable wireless systems to achieve high spectral efficiency (SE) while maintaining a target quality of service and are rising a great interest for high data rate communications in fading channels. In this paper, we analyze slow adaptive M-ary quadrature amplitude modulation systems with diversity subject to interference. We derive the performance in terms of mean SE and bit error outage for systems experiencing co-channel interference with optimum combining. The analysis leads to a new definition of in-service regions, to be used for the optimum constellation size selection in place of signal-to-noise ratio thresholds.**

I. Introduction

Adaptive modulation techniques allow to maximize the spectral efficiency (SE) in faded channels without compromising the performance in terms of bit error probability (BEP) and bit error outage (BEO) (see, e.g., [1]–[3]). Fast adaptive modulation (FAM) techniques [2] track the small-scale fading leading to best performance, at the cost of a frequent channel estimation or prediction and feedback. With respect to FAM, a gain in terms of reduced feedback rate and complexity can be achieved by slow adaptive modulation (SAM) techniques [3], that adapt modulation parameters to the channel variations averaged over the small-scale fading (i.e., tracking large-scale fading). In [3], it was shown that SAM performance is close to FAM and achieves a relevant improvement of SE and BEO with respect to non-adaptive modulation schemes.

For multiuser systems, a possible cause of performance degradation is the presence of unexpected interfering signals. Orthogonal frequency-division multiplexing (OFDM) systems with FAM in the presence of interference were investigated in [4], where the performance is quantified in terms of bit error rate. In [5], [6] the performance of interfered M-ary phase shift keying (M-PSK) systems is derived, while in [7], [8] the validity of the Gaussian approximation for the interference is verified.

In this paper, we analyze the effects of co-channel interference on the required signal-to-noise ratio (SNR) thresholds for slow adaptive M-ary quadrature amplitude modulation (M-QAM) systems employing the optimum combiner (OC) [9] at the receiver. First, the analysis is performed conditioned to a given level of co-channel interference. Then a more general case is investigated assuming that, due to the shadowing level, both the SNR and the interference-to-noise ratio (INR) are independent random variables (RVs). This leads to the introduction of a new concept of in-service regions based on signal-to-noise-plus-interference ratio (SINR) for a target BEP in a given system configuration.

The remainder of this paper is organized as follows. In Section II, the system model is described and in Section III the BEP expression is obtained for SAM with diversity under interference. Then, in Section IV, the performance metrics are defined and evaluated in terms of BEO and mean SE. Finally, numerical results are provided in Section VI, and conclusions are given in Section V.

Notation: throughout the paper, vectors and matrices are indicated by bold; det$[\mathbf{M}]$ denotes the determinant of the matrix $\mathbf{M}$; the superscript ^{H} indicates the conjugation and transposition operation, and $\mathbf{I}_N$ is the $N \times N$ identity matrix.

II. System Model

We assume M-QAM constellation signaling over composite Rayleigh fading and log-normal shadowing, and N-branches multiple channels reception. Independent, identically distributed (i.i.d.) fading and same shadowing level over all branches (i.e., microdiversity) are considered. The optimum combining technique is employed to process signals over branches [9] as it will be detailed at the end of the section. In particular, the minimum mean square error (MMSE) and maximum ratio combining (MRC) techniques are considered for interfered and uninterfered cases, respectively.

We denote by E_s the mean (averaged over the fading) transmitted symbol energy, by $\mathbf{h} = [h_1, h_2, \ldots, h_N]^T$ the channel vector, whose elements h_i represent fading gain for the i-th channel and are complex Gaussian RVs with $\mathbb{E}\{h\} = 0$ and $\mathbb{E}\{|h|^2\} = 1$ (i.e., $h \sim \mathcal{CN}(0,1)$), and by $\mathbf{n}(k)$ the additive white Gaussian noise (AWGN) with mean zero and $\mathbb{E}\{\mathbf{n}(k)\mathbf{n}(k)^H\} = N_0 \mathbf{I}_N$, where N_0 is the one-sided thermal noise power spectral density. The mean SNR per branch is[1] $\overline{\gamma} = \mathbb{E}\{|h|^2\}E_\text{s}/N_0 = E_\text{s}/N_0$. We consider a log-normal shadowing, thus $\overline{\gamma}_\text{dB} = 10\log_{10}(\overline{\gamma})$ is a Gaussian distributed RV with mean μ_dB and variance σ^2_dB.

[1]Since we consider microdiversity, in the following we will omit the branch subscript in the mean SNR notation.

Adaptive modulation techniques allow to achieve the best SE accordingly with channel conditions, minimizing the outage probability [3]. In particular, a set of $J+1$ constellation sizes $\{M_0, M_1, \ldots, M_J\}$ can be adopted. For uninterfered SAM systems, the optimal constellation size is chosen depending on the mean SNR. When, even with the lowest constellation size M_0, $\overline{\gamma}$ is not sufficient to guarantee the target BEP $P_{\text{b}}^{\star}$, the system is in outage. The constellation size is chosen comparing the $\overline{\gamma}$ value with thresholds, that are set to guarantee the target BEP when the j-th constellation size is adopted, that is $P_{\text{b}}(\overline{\gamma}_j^{\star}) = P_{\text{b}}^{\star}$. In particular, when the SNR value falls within the j-th region $(\overline{\gamma}_j^{\star}, \overline{\gamma}_{j+1}^{\star}]$, the j-th constellation size M_j is adopted.

In the presence of N_{I} interfering signals, the received signal on the generic i-th branch after matched filtering and sampling at the symbol rate is given by $z_i(k) = z_i^{(\text{P})}(k) + j z_i^{(\text{Q})}(k)$, with k representing the time index, and j the imaginary unit. Note that the superscripts (P) and (Q) stand for the in-phase and quadrature components, respectively. The signal at the output of the receiving antennas $\mathbf{z}(k) = [z_1(k), \ldots, z_N(k)]^H$ can be written as [6]

$$\mathbf{z}(k) = \sqrt{E_{\text{S}}}\mathbf{h}\; b_0(k) + \sqrt{E_{\text{I}}} \sum_{n=1}^{N_{\text{I}}} \mathbf{h}_{\text{I},n} b_{\text{I},n}(k) + \mathbf{n}(k) \qquad (1)$$

where E_{I} is the mean symbol energy transmitted by each interfering signals, $b_0(k) = b_0^{(\text{P})}(k) + j b_0^{(\text{Q})}(k)$ is the useful signal sample, and $b_{\text{I},n}(k) = b_{\text{I},n}^{(\text{P})}(k) + j b_{\text{I},n}^{(\text{Q})}(k)$ is the interfering signal sample from the n-th interferer. We assume a slow frequency flat Rayleigh distributed fading. The vectors $\mathbf{h}$ and $\mathbf{h}_{\text{I},n}$, with $n = 1, 2, \ldots, N_{\text{I}}$ have distribution $\mathcal{CN}(0, \mathbf{I}_N)$, and we denote by $\mathbf{H}_{\text{I}} = \{\mathbf{h}_{\text{I,j}}\}_{j=1,\ldots,N_{\text{I}}}$ the interference channel matrix, where N_{I} is the number of interfering signals. Note that $b_1(k), \ldots, b_{N_{\text{I}}}(k)$ are independent zero-mean complex Gaussian RVs, each with unitary variance [7].

As far as the optimum combining technique is concerned, we recall that the optimum weight vector to combine signals over branches and maximize the SINR is given by

$$\mathbf{w} = \alpha \mathbf{R}^{-1} \mathbf{h} \qquad (2)$$

where α is an arbitrary constant and $\mathbf{R}$ the covariance matrix given by

$$\begin{aligned}\mathbf{R} &= \mathbb{E}_{\mathbf{n}(k), b_j(k)} \left\{ \left[\sqrt{E_{\text{I}}} \sum_{n=1}^{N_{\text{I}}} \mathbf{h}_{\text{I},n} b_{\text{I},n}(k) + \mathbf{n}(k) \right] \right. \\ &\quad \left. \times \left[\sqrt{E_{\text{I}}} \sum_{n=1}^{N_{\text{I}}} \mathbf{h}_{\text{I},n} b_{\text{I},n}(k) + \mathbf{n}(k) \right]^H \right\} \\ &= \underbrace{E_{\text{I}} \sum_{j=1}^{N_{\text{I}}} \mathbf{h}_{\text{I},j} \mathbf{h}_{\text{I},j}^H}_{\mathbf{R}_I} + N_0 \mathbf{I}_N \end{aligned} \qquad (3)$$

where the last equality holds because the desired signal, the interference and the noise are assumed mutually independent. The components of the received vector $\mathbf{z}(k)$ are combined through the vector of the optimum weight $\mathbf{w}^H$ giving

$$r(k) = \mathbf{w}^H \mathbf{z}(k)\,.$$

In the absence of interference, $\mathbf{w} = \mathbf{h}$ and the OC reduces to the well known MRC [10], [11].

III. BEP Evaluation

In this section, we derive the BEP for M-QAM systems with optimum combining at the receiver. In particular, we extend to M-QAM systems the methodology applied in [5], [6] for M-PSK signals.

We consider an overloaded system[2] (i.e., $N_{\text{I}} \geq N$) for which the SINR at the combiner output can be written as $\gamma_{\text{T}} = E_{\text{s}} \mathbf{h}^H \mathbf{R}^{-1} \mathbf{h}$. We denote the instantaneous SINR by γ_{T}, the instantaneous INR by γ_{I}, and the instantaneous SNR by γ. The matrix $\mathbf{R}^{-1}$ can be written as $\mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^H$, where $\mathbf{U}$ is a unitary matrix with elements $[\mathbf{u}_1, \ldots, \mathbf{u}_N]$, and $\mathbf{\Lambda}$ is a diagonal matrix whose elements on the principal diagonal are the eigenvalues of $\mathbf{R}$, denoted by $\widetilde{\lambda}_1, \ldots, \widetilde{\lambda}_N$. Then the SINR is

$$\gamma_{\text{T}} = E_{\text{s}} \mathbf{h}^H \mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^H \mathbf{h} = \sum_{i=1}^{N} \frac{E_{\text{s}} \left| \mathbf{h}^H \mathbf{u}_i \right|^2}{E_{\text{I}} \lambda_i + N_0} \qquad (4)$$

where $\mathbf{h}^H \mathbf{u}_1, \ldots, \mathbf{h}^H \mathbf{u}_N$ have the same distribution of $\mathbf{h}$, being $\mathbf{U}$ a unitary transformation, and the eigenvalues of $\mathbf{R}$ have been written in terms of the eigenvalues of $\mathbf{R}_{\text{I}}$, $\{\lambda_i\}$, as

$$\widetilde{\lambda}_i = E_{\text{I}} \lambda_i + N_0\,, \qquad i = 1, 2, \ldots, N\,.$$

Since H_{I} is a matrix whose elements are complex Gaussian distributed $\mathcal{CN}(0,1)$, then the matrix $\mathcal{W} = H_{\text{I}} H_{\text{I}}^H = \mathbf{R}_{\text{I}}$ is a central Wishart matrix [12], with eigenvalues having distribution given by

$$f_{\boldsymbol{\lambda}}(x_1, \ldots, x_N) = \frac{1}{N!} \prod_{i=1}^{N} \frac{x_i^{N_{\text{I}}-N} e^{-x_i}}{(N-i)!(N_{\text{I}}-i)!} \; \prod_{i<j}^{N} (x_i - x_j)^2\,. \qquad (5)$$

Starting from the exact instantaneous BEP [3], [13] the mean BEP for M-QAM systems can be derived averaging over the SINR:

$$\begin{aligned} P_{\text{b}}\left(\overline{\gamma}_{\text{T}}\right) &= \int P(e|\gamma_{\text{T}}) f_{\gamma_{\text{T}}|\overline{\gamma}_{\text{T}}}(\xi) d\xi \\ &= \frac{2}{\sqrt{M} \log_2(\sqrt{M})} \sum_{h=1}^{\log_2(\sqrt{M})} \sum_{n=0}^{(1-2^{-h})\sqrt{M}-1} (-1)^{\left\lfloor \frac{n 2^{h-1}}{\sqrt{M}} \right\rfloor} \\ &\quad \times \left(2^{h-1} - \left\lfloor \frac{n 2^{h-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \zeta_n(\gamma_{\text{T}}) \end{aligned} \qquad (6)$$

where the last equality is due to the use of the Craig's formula of the Gaussian Q function [11], $\Psi_{\gamma_{\text{T}}}\{j\nu\} \triangleq \mathbb{E}\{e^{j\nu\gamma_{\text{T}}}\}$ is the characteristic function (c.f.) of γ_{T}, $\overline{\gamma}_{\text{T}}$ is the mean SINR, and

$$\zeta_n(\gamma_{\text{T}}) = \frac{1}{\pi} \int_0^{\pi/2} \Psi_{\gamma_{\text{T}}} \left\{ -\frac{3(2n+1)^2}{2(M-1)\sin^2\theta} \right\} d\theta$$

[2]In this paper, for sake of brevity, we consider the overloaded case only, but the analysis can be extended to the underloaded case.

By using the chain rule of conditional expectation, we obtain

$$\zeta_n(\gamma_{\rm T}) = \frac{1}{\pi}\int_0^{\pi/2}\int_0^{\infty}\cdots\int_0^{\infty}\Psi_{\gamma_{\rm T}|\boldsymbol{\lambda}}\left\{-\frac{c_{M,n}}{\sin^2\theta}\right\} f_{\boldsymbol{\lambda}}(\mathbf{x})d\theta d\mathbf{x} \tag{7}$$

where $c_{M,n}=\frac{3(2n+1)^2}{2(M-1)}$. Details on the evaluation of (7) are provided in the Appendix, from which $\zeta_n(\gamma_{\rm T})$ results to be given by

$$\begin{aligned}\zeta_n(\gamma_{\rm T}) &= \frac{1}{\pi}\frac{1}{\prod_{i=1}^{N}(N-i)!(N_{\rm I}-i)!}\\ &\times\int_0^{\pi/2}\det\left(\left\{b^{(i+j+N_{\rm I}-N)}e^b(i+j+N_{\rm I}-N)!\right.\right.\\ &\times\ [b(1+(i+j+N_{\rm I}-N))\Gamma(-1-i-j-N_{\rm I}+N,b)\\ &+\ a\Gamma(-(i+j+N_{\rm I}-N),b)]\}_{i,j=0,\dots,N-1}\Big)\,d\theta. \end{aligned}\tag{8}$$

Substituting (8) in (6), the mean BEP expression becomes (9), reported at the bottom of the page, where $\mathcal{G}^{(n)}=\{G^{(n)}_{i+j+N_{\rm I}-N}\}_{i,j=0,\dots,N-1}$ is an Hankel matrix with elements given by (10) and with $\overline{\gamma}_I=(N_I E_I)/N_0$ denoting the mean INR. In the absence of interference, the SNR at the combiner output γ results

$$\gamma=\sum_{i=1}^{N}|h_i|^2\frac{E_{\rm s}}{N_0}. \tag{11}$$

In this case, for M-QAM systems with ideal channel estimates, no interference, and MRC at the receiver, the mean BEP is derived from (6), with c.f. $\Psi_\gamma(j\nu)=(1-j\nu\overline{\gamma})^{-N}$, and is given in [14].

IV. Metrics for SAM Systems

We now evaluate the performance of both interfered and uninterfered systems in terms of SE and BEO. As already observed, in order to select the optimal constellation size, and in order to properly evaluate the mean SE and BEO, the knowledge of the SINR thresholds is required. Note that, from (9), the BEP for interfered systems is a function of both the SNR and INR, thus of the SINR. In addition, since also the interference signal is affected by the shadowing, $\overline{\gamma}_{\rm I}$ is a RV that we model as a log-normal RV, that is $\overline{\gamma}_{\rm I,dB}\sim\mathcal{N}(\mu_{\rm I,dB},\sigma^2_{\rm dB})$ [15], [16]. We first evaluate the performance conditioned to a given level of signal-to-interference ratio (SIR), that is defined on the total amount of interference as $\text{SIR}=E_{\rm s}/(N_{\rm I}E_{\rm I})=\overline{\gamma}/\overline{\gamma}_{\rm I}$.[3] For a given SIR value, the SNR thresholds $\breve{\gamma}_j^\star$ are evaluated from (9) as the value such that $P_{\rm b|SIR}(\breve{\gamma}_j^\star)=P_{\rm b}^\star$. We recall that the interference affects the system, leading to a worse BEP performance. It follows that a SNR thresholds shifting occurs, that is $\Delta j_{\rm dB}=\breve{\gamma}^\star_{j,\rm dB}\left(\overline{\gamma}_{\rm I,dB}\right)-\overline{\gamma}^\star_{j,\rm dB}\geq 0$, where $\breve{\gamma}^\star_{j,\rm dB}\left(\overline{\gamma}_{\rm I,dB}\right)$ is the required SNR value to reach the target BEP when the mean INR equals $\overline{\gamma}_{\rm I,dB}$, while $\overline{\gamma}^\star_{j,\rm dB}$ is the required SNR in the absence of interference.

An important figure of merit that can be evaluated conditioned to the SIR value is the *bit error outage*, i.e., the probability that the BEP is greater than the target BEP,

$$P_{\rm o|SIR}(P_{\rm b}^\star)=\mathbb{P}\left\{P_{\rm b}(\overline{\gamma})>P_{\rm b}^\star\right\}. \tag{12}$$

The adaptive system is in outage when even the more robust modulation scheme (M_0) does not satisfy the target BEP, thus

$$P_{\rm o|SIR}(P_{\rm b}^\star)=F_{\overline{\gamma}}(\breve{\gamma}_0^\star)=F_{\overline{\gamma}_{\rm dB}}\left(\breve{\gamma}^\star_{\rm dB,0}\right) \tag{13}$$

where $F_{\overline{\gamma}_{\rm dB}}(x)$ is the CDF of the mean SNR expressed as

$$F_{\overline{\gamma}_{\rm dB}}(\xi)=Q\left(\frac{\mu_{\rm dB}-\xi}{\sigma_{\rm dB}}\right) \tag{14}$$

Another important measure of the QoS is the *mean spectral efficiency* [bps/Hz] which is defined as

$$\begin{aligned}\eta_{|\rm SIR} &= \sum_{j=0}^{J-1}\tilde{M}_j\ \mathbb{P}\left\{\breve{\gamma}_j^\star<\overline{\gamma}\leq\breve{\gamma}_{j+1}^\star\right\}+\tilde{M}_J\ \mathbb{P}\left\{\breve{\gamma}_J^\star<\overline{\gamma}\right\}\\ &= \sum_{j=0}^{J-1}\tilde{M}_j\ \left[F_{\overline{\gamma}}(\breve{\gamma}_{j+1}^\star)-F_{\overline{\gamma}}(\breve{\gamma}_j^\star)\right]+\tilde{M}_J\ \left[1-F_{\overline{\gamma}}(\breve{\gamma}_J^\star)\right]\end{aligned} \tag{15}$$

where $\tilde{M}_j=\log_2 M_j$.

We now consider the case where the shadowing levels on the useful and the interfering signals are assumed independent. Then, the BEP expression is a function of two RVs, the INR and the SNR $(\overline{\gamma}_{\rm I},\overline{\gamma})$. Hence, the SINR in-service region for which the target BEP for a given system configuration is

[3] Conditioning the system to the SIR value implies that, for further evaluation analysis (i.e., BEO or SE), the interference shadowing level follows the useful signal shadowing level. It means that, $\overline{\gamma}_{\rm I}$, rather than being a RV independent from $\overline{\gamma}$, is a function of the $\overline{\gamma}$. In particular, $\overline{\gamma}_{\rm I}=\overline{\gamma}/\text{SIR}$.

$$\begin{aligned}P_{\rm b}\left(\overline{\gamma}_{\rm T}\right)=P_{\rm b}\left(\overline{\gamma},\overline{\gamma}_{\rm I}\right) &= \frac{2}{\sqrt{M}\log_2(\sqrt{M})}\sum_{h=1}^{\log_2(\sqrt{M})}\ \sum_{n=0}^{(1-2^{-h})\sqrt{M}-1}(-1)^{\left\lfloor\frac{n2^{h-1}}{\sqrt{M}}\right\rfloor}\\ &\times\left(2^{h-1}-\left\lfloor\frac{s2^{h-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right)\left[\pi\prod_{i=1}^{N}(N-i)!(N_I-i)!\right]^{-1}\int_0^{\pi/2}\det\mathcal{G}^{(n)}(\theta,\overline{\gamma},\overline{\gamma}_{\rm I})d\theta\,,\end{aligned} \tag{9}$$

$$\begin{aligned}G_k^{(n)}(\theta,\overline{\gamma},\overline{\gamma}_{\rm I}) &= \left(\frac{c_{M,n}N_{\rm I}\overline{\gamma}}{\sin^2\theta\ \overline{\gamma}_{\rm I}}+\frac{N_{\rm I}}{\overline{\gamma}_{\rm I}}\right)^k\exp\left(\frac{c_{M,n}N_{\rm I}\overline{\gamma}}{\sin^2\theta\ \overline{\gamma}_{\rm I}}+\frac{N_{\rm I}}{\overline{\gamma}_{\rm I}}\right)k!\\ &\times\left[\left(\frac{c_{M,n}N_{\rm I}\overline{\gamma}}{\sin^2\theta\ \overline{\gamma}_{\rm I}}+\frac{N_{\rm I}}{\overline{\gamma}_{\rm I}}\right)(1+k)\Gamma\left(-1-k,\frac{c_{M,n}N_{\rm I}\overline{\gamma}}{\sin^2\theta\ \overline{\gamma}_{\rm I}}+\frac{N_{\rm I}}{\overline{\gamma}_{\rm I}}\right)\frac{N_{\rm I}}{\overline{\gamma}_{\rm I}}\Gamma\left(-k,\frac{c_{M,n}N_{\rm I}\overline{\gamma}}{\sin^2\theta\ \overline{\gamma}_{\rm I}}+\frac{N_{\rm I}}{\overline{\gamma}_{\rm I}}\right)\right],\end{aligned} \tag{10}$$

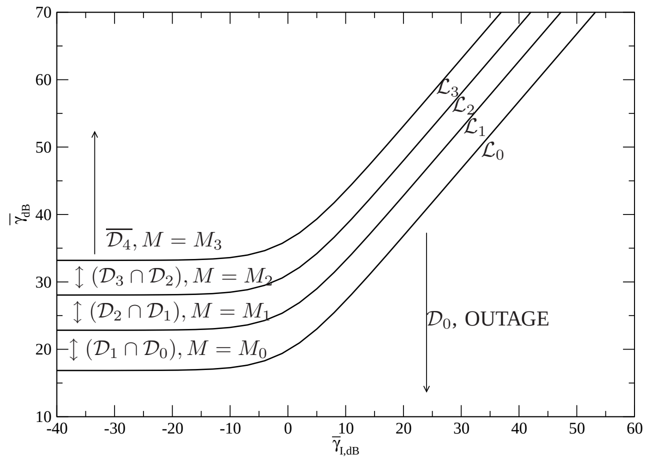

Figure 1. In-service and outage regions for M-QAM systems.

satisfied is related to the required couples $(\overline{\gamma}_{\rm I}, \overline{\gamma})$ as depicted in Fig. 1, where the behavior of the SINR in-service regions is reported for M-QAM systems. In the figure, the in-service region for the generic j-th constellation size (when a given diversity technique is adopted) is limited by the locus of the couples $(\overline{\gamma}_{\rm I}, \overline{\gamma})$ for which $P_{\rm b} = P_{\rm b}^\star$, when the M_j modulation is adopted, and it is denoted by

$$\mathcal{L}_j = \{(\overline{\gamma}_{\rm I}^\star, \overline{\gamma}^\star) \quad \text{s.t.} \quad P_{\rm b} = P_{\rm b}^\star \quad \text{with} \quad M = M_j\} \, .$$

For each value $(\overline{\gamma}_{\rm I}, \overline{\gamma})$ above this line the system is in-service, while for each $(\overline{\gamma}_{\rm I}, \overline{\gamma})$ below it the system is in outage. We denote by $\mathcal{D}_j$ the outage region when the M_j modulation is adopted, and by $\overline{\mathcal{D}_j} = \mathbb{R}_2 - \mathcal{D}_j$ the complementary region of $\mathcal{D}_j$, that is the in-service region for the j-th constellation size. In particular, the outage region can be expressed as

$$\begin{aligned}\mathcal{D}_j &\triangleq \{(\overline{\gamma}_{\rm I}, \overline{\gamma}) \quad \text{s.t.} \quad P_{\rm b} > P_{\rm b}^\star \quad \text{with} \quad M = M_j\} \\ &= \{(\overline{\gamma}_{\rm I}, \overline{\gamma}) \quad \text{s.t.} \quad \overline{\gamma}_{\rm I} < \overline{\gamma}_{\rm I}^\star \quad \text{or} \quad \overline{\gamma} < \overline{\gamma}^\star\} \\ &= \left\{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB}) \quad \text{s.t.} \quad \overline{\gamma}_{\rm I,dB} < \overline{\gamma}_{\rm I,dB}^\star \quad \text{or} \quad \overline{\gamma}_{\rm dB} < \overline{\gamma}_{\rm dB}^\star\right\} \, .\end{aligned} \tag{16}$$

Note that, due to the decrease of the interference signal power, the locus of $(\overline{\gamma}_{\rm I}^\star, \overline{\gamma}^\star)$ experiences a floor behavior for low INR values. In particular, the lines merge to the SNR threshold value $\overline{\gamma}^\star$ of the uninterfered system.

In the considered system, when the couple $(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB})$ falls within the region $(\mathcal{D}_{j+1} \cap \mathcal{D}_j)$, the j-th modulation is adopted. When $(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB}) \in \mathcal{D}_0$, the target BEP can not be reached and the system is in outage. Thus, the BEO can be defined as

$$P_{\rm o} = \mathbb{P}\left\{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB}) \in \mathcal{D}_0\right\} = F_{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB})}(\mathcal{D}_0) \, . \tag{17}$$

Since the SNR and INR are log-normal RVs, the joint cumulative distribution function (CDF) can be expressed as

$$\begin{aligned}F_{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB})}(\mathcal{D}_j) &= \mathbb{P}\left\{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB}) \in \mathcal{D}_j\right\} \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\breve{\gamma}_{j,\rm dB}^\star(\overline{\gamma}_{\rm I,dB})} f_{\overline{\gamma}_{\rm dB}}(\xi) f_{\overline{\gamma}_{\rm I,dB}}(\xi_{\rm I}) d\xi d\xi_{\rm I} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Q\left(\frac{\mu_{\rm dB} - \breve{\gamma}_{j,\rm dB}^\star}{\sigma_{\rm dB}}\right) \\ &\quad \times \exp\left[-\frac{(\xi_{\rm I} - \mu_{\rm I,dB})^2}{\sqrt{2}\sigma_{\rm dB}}\right] d\xi_{\rm I} \, .\end{aligned} \tag{18}$$

where $f_{\overline{\gamma}_{\rm I,dB}}(\cdot)$ and $f_{\overline{\gamma}_{\rm dB}}(\cdot)$ are the probability density functions (PDFs) of the INR and SNR, respectively.

For what concerns the mean SE, it is given by (19) at the bottom of the page. In the absence of interference, the in-service regions becomes the set of SNR values such that $\overline{\gamma}_j > \overline{\gamma}_j^\star$, that is $\mathcal{L}_j$ reduces to $\overline{\gamma}_j^\star$, and $F_{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB})}(\mathcal{D}_j)$ merges to $F_{\overline{\gamma}_{\rm dB}}(\overline{\gamma}_{\rm dB}^\star) = \mathbb{P}\{\overline{\gamma}_{\rm dB} \leq \overline{\gamma}_{\rm dB}^\star\}$. In this case, the BEO and SE can be obtained from (13) and (15), where the thresholds $\breve{\gamma}_{\rm dB}^\star$ are substituted by the ideal ones $\overline{\gamma}_{\rm dB}^\star$.

In both the cases (ideal or interfered systems), the analysis of the performance in terms of BEO and SE needs the inverse BEP expression which requires, in general, numerical root evaluation in $\overline{\gamma}_{\rm I}$ and $\overline{\gamma}$.

V. Numerical Results

We now present numerical results in terms of mean SE and BEO for SAM systems in the presence of co-channel interference. Coherent detection of M-QAM with optimum combining and Gray code mapping in composite Rayleigh fading and log-normal shadowing channels with ideal channel estimations is considered. First, results conditioned to a given value of SIR are provided, then unconditioned results are reported showing the SINR in-service region and the performance for SAM systems. As far as the conditioned case is concerned, the SNR thresholds are evaluated as the SNR values that achieve a target BEP of $P_{\rm b} = 10^{-2}$ (that is a typical value for uncoded systems) for different constellation sizes with a maximum BEO of 5%. The BEP expression employed is (9), while the BEO and mean SE are (13) and (15), respectively.

In Fig. 2, the BEP as a function of the mean SNR for a fixed 64-QAM system is reported for different numbers of diversity branches, interfering signals, and SIR values. As expected, the curves show a floor for increasing mean SNR (interference-limited systems). The lower is the SIR, the higher is the BEP floor value. Moreover, some system configurations (i.e.,

$$\begin{aligned}\eta &= \sum_{j=0}^{J-1} \tilde{M}_j \; \mathbb{P}\left\{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB}) \in (\mathcal{D}_{j+1} \cap \mathcal{D}_j)\right\} + \tilde{M}_J \; \mathbb{P}\left\{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB}) \in \overline{\mathcal{D}}_J\right\} \\ &= \sum_{j=0}^{J-1} \tilde{M}_j \; \left[F_{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB})}(\mathcal{D}_{j+1}) - F_{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB})}(\mathcal{D}_j)\right] + \tilde{M}_J \; \left[1 - F_{(\overline{\gamma}_{\rm I,dB}, \overline{\gamma}_{\rm dB})}(\mathcal{D}_J)\right]\end{aligned} \tag{19}$$

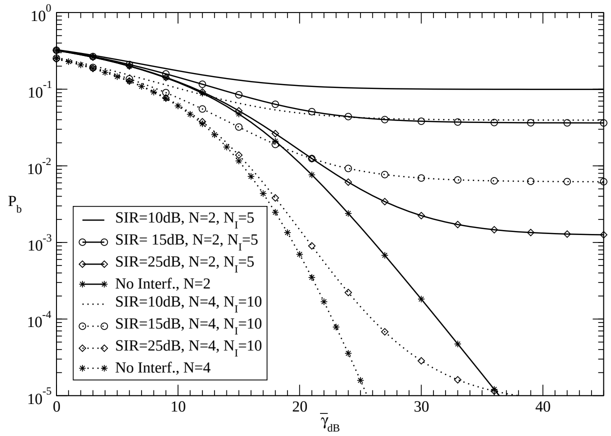

Figure 2. BEP vs. $\overline{\gamma}_{\mathrm{dB}}$ for 64-QAM systems. The BEP, conditioned to SIR, is evaluated for several N and N_{I} values.

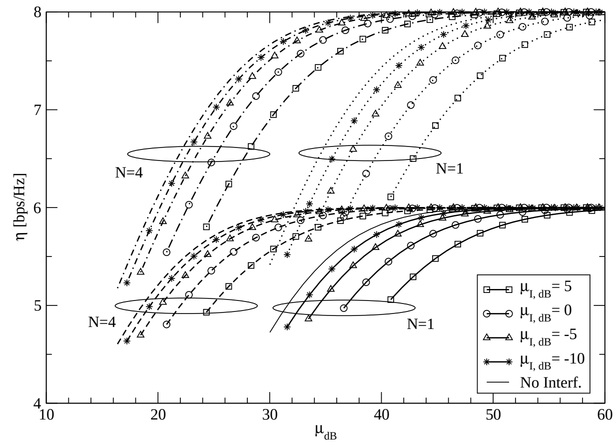

Figure 4. Mean spectral efficiency vs. μ_{dB} SAM systems with $M_{\max} = 64$ and 256, $N = 1$ and 4, $N_{\mathrm{I}} = 5$, BEO $< 5\%$, $P_{\mathrm{b}}^{\star} = 10^{-2}$, and $\sigma_{\mathrm{dB}} = 8$.

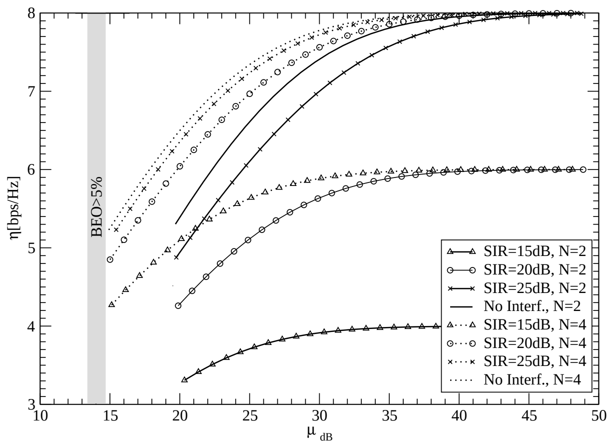

Figure 3. Mean SE (conditioned to SIR) vs. μ_{dB} for SAM systems with $M_{\max} = 256$, BEO $< 5\%$, $P_{\mathrm{b}}^{\star} = 10^{-2}$, $\sigma_{\mathrm{dB}} = 8$, $N_{\mathrm{I}} = 6$, and $N = 2$, and 4.

some N_{I} or N values) might not overcome the presence of interference, and the system experiences BEP greater than the target BEP 10^{-2}. It follows that the system might not switch to that constellation size, even with high SNR.

This behavior can be translated in terms of mean SE, and observed in Fig. 3. Here, the mean SE as a function of μ_{dB} is provided for SAM systems with $N_{\mathrm{I}} = 6$, different diversity orders N, and different SNRs. The system can switch the constellation size M in $\{4, 16, 64, 256\}$ and the expected maximum SE is 8 bps/Hz. It is worthwhile noting the extremely low performance of the system when $N = 2$ and SIR $= 15$dB. In this case, $M = 4, 16$ are actually the only values that satisfy the target BEP of 10^{-2}. By increasing the SIR value or the diversity of the system, the mean SE increases, reaching values close to the SE of the uninterfered system.

We now provide results for the unconditioned case. In particular, the BEP is given by (9), the BEO and mean SE are expressed in (17) and (19), respectively. The SINR in-service regions are reported in Fig. 1 for systems with $M \in \{4, 16, 64, 256\}$, $P_{\mathrm{b}}^{\star} = 10^{-2}$, $\sigma_{\mathrm{dB}} = 8$, $N = 2$, and $N_{\mathrm{I}} = 5$.

The mean SE vs. μ_{dB} can be observed in Fig. 4 for SAM systems with $M_{\max} = 64$ and 256, $N = 1$, $N_{\mathrm{I}} = 5$, and several $\mu_{\mathrm{I,dB}}$ values ($\mu_{\mathrm{I,dB}} = -10, -5, 0, 5$). Note that, by decreasing the interference level (i.e., by lowering $\overline{\gamma}_{\mathrm{I}}$), the SE merges to the one obtained from the uninterfered system. Moreover, the higher the diversity order (i.e., N) the lower is the gap between the $\mu_{\mathrm{I,dB}} = -10$ dB curve and the ideal one. In fact, for the system with $N = 4$, the in-service regions achieve the floor for values of $\mu_{\mathrm{I,dB}}$ lower than the system without diversity.

From the above results, given a diversity technique, a constellation size set, and the amount of interference, the system designer can obtain the minimum value of μ_{dB} for specified target BEP, BEO, and SE. Since the actual μ_{dB} is tied to the propagation law and location of the user, one can design the wireless system (e.g., cell size, and power levels for cellular systems) that fulfills the given requirements.

VI. Conclusion

In this paper, slow adaptive M-ary quadrature amplitude modulation systems with optimum combining diversity in composite small- and large-scale fading have been analyzed in the presence of co-channel interference. In order to understand the sensitivity to system parameters, channel conditions, and amount of interference, we investigated the signal-to-noise-plus-interference ratio in-service regions and the performance in terms of bit error outage and mean spectral efficiency. The framework enables the derivation of performance metrics for several systems configurations, providing a useful tool for wireless systems design given specified requirements.

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Appendix
Evaluation of $\zeta_n(\gamma_{\rm T})$

Since the vector $[\mathbf{h}^H\mathbf{u}_1, \ldots, \mathbf{h}^H\mathbf{u}_N]$ is Gaussian with i.i.d. elements, the conditional characteristic function of γ is given by

$$\Psi_{\gamma_{\rm T}|\boldsymbol{\lambda}}(j\nu) = \prod_{i=1}^{N}\left(1 - j\nu\frac{E_{\rm S}}{E_{\rm I}\lambda_i + N_0}\right)^{-1} \qquad (20)$$

Substituting (20) in (7), we obtain

$$\begin{aligned}\zeta_n(\gamma_{\rm T}) = & \frac{1}{\pi N!}\frac{1}{\prod_{i=1}^{N}(N-i)!(N_{\rm I}-i)!} \\ & \times \int_0^{\pi/2}\int_0^{\infty}\ldots\int_0^{\infty}|\mathbf{V}_1(\mathbf{x})|^2 \\ & \times \prod_{i=1}^{N}\frac{\sin^2\theta}{\sin^2\theta + c_{M,n}\frac{E_{\rm S}}{E_{\rm I}x_i+N_0}}e^{-x_i}x_i^{N_{\rm I}-N} \qquad (21)\end{aligned}$$

where $\mathbf{V}_1(\mathbf{x})$ is the Vandermonde matrix and $|\mathbf{V}_1(\mathbf{x})|^2 = \prod_{i<j}^{N}(x_i - x_j)^2$. Recalling the Lemma 1 in [17]

$$\begin{aligned}&\int_0^{\infty}\ldots\int_0^{\infty}|\mathbf{\Psi}(\mathbf{x})||\mathbf{\Phi}(\mathbf{x})|\prod_{k=1}^{K}\xi(x_i)dx_1\ldots dx_K \\ &= K!\det\left(\left\{\int_0^{\infty}\Phi_i(x)\Psi_j(x)\xi(x)dx\right\}_{k,j=1,\ldots,K}\right) \qquad (22)\end{aligned}$$

considering $|\mathbf{\Psi}(\mathbf{x})| = |\mathbf{\Phi}(\mathbf{x})| = |\mathbf{V}_1(\mathbf{x})|$, $K = N$ and

$$\xi(x) = \frac{\sin^2\theta}{\sin^2\theta + c_{M,n}\frac{E_{\rm D}}{E_{\rm I}x_i+N_0}} \quad e^{-x_i}x_i^{N_{\rm I}-N}$$

we obtain

$$\begin{aligned}\zeta_n(\gamma_{\rm T}) = & \frac{1}{\pi}\frac{1}{\prod_{i=1}^{N}(N-i)!(N_{\rm I}-i)!} \\ & \times\int_0^{\pi/2}\det\Bigg(\Bigg\{\int_0^{\infty}e^{-x}x^{N_{\rm I}-N+j+i-2} \\ & \times\left(c_{M,n}\frac{E_{\rm D}}{E_{\rm I}x_i+N_0}\right)dx\Bigg\}_{i,j=1,\ldots,N}\Bigg)d\theta\,. \qquad (23)\end{aligned}$$

We apply the following identity

$$\begin{aligned}&\int_0^{\infty}e^{-x}x^n\frac{x+a}{x+b}dx = \\ &\quad = b^n e^b n![b(1+n)\Gamma(-1-n,b) + a\Gamma(-n,b)] \qquad (24)\end{aligned}$$

that is valid for $n > -1$ and $\arg b \neq \pi$, where

$$\begin{aligned}a &= \frac{N_0}{E_{\rm I}} = \frac{N_{\rm I}}{\overline{\gamma}_{\rm I}}, \\ b &= \frac{E_{\rm D}}{N_0} + \frac{E_{\rm d}}{E_{\rm I}}\frac{c_{M,n}}{\sin^2\theta} = \frac{N_{\rm I}}{\overline{\gamma}_{\rm I}} + \frac{N_{\rm I}\overline{\gamma}}{\overline{\gamma}_{\rm I}}\frac{c_{M,n}}{\sin^2\theta} \\ n &= N_{\rm I} - N + i + j - 2\,.\end{aligned}$$

Then, the expression in (23) results in (8) reported in Section III.

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