On the Design of Space-Time Trellis Codes for Cooperative Relaying

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***Abstract*—The design of space-time codes for wireless communications with relays is investigated by considering a pragmatic approach based on the concatenation of convolutional codes and BPSK/QPSK modulation to obtain cooperative codes. We propose a design criterion which aims at optimizing both diversity and coding gain, based on an asymptotic bound for frame error probability. This framework is useful to characterize the behavior of Cooperative Pragmatic Space-Time Codes (CP-STC) and to set up a code search procedure to obtain good CP-STC with overlay construction (COP-STC) which are suitable for cooperative communication with a variable number of relays and have improved performance with respect to cooperative codes in the literature. The CP-STCs result to perform quite well in block fading channels, including quasi-static channel, even with a low number of states and relays. Moreover, their implementation is low complexity, since it requires common convolutional encoders and Viterbi decoders with suitable generators and rates.**

I. Introduction

Cooperative communications are a new paradigm involving both transmission and distributed processing which promises significant increase of capacity and diversity gain in wireless networks, by counteracting faded channels with cooperative diversity. In addition to physical antenna arrays, the relay channel model enables the exploitation of distributed antennas belonging to multiple relaying terminals. This form of space diversity if referred to as *cooperative diversity* because terminals share antennas and other resources to create a virtual array through distributed transmission and signal processing. Several issues are arising with the aim to exploit cooperative diversity such as, among others, channel modeling and implementation aspects [1], protocols and resource management [2], the choice of proper relays [3], power allocation among cooperating nodes [4] and cooperative/distributed space-time codes (STC) [5], [6]. This work is devoted to this latter aspect.

With the introduction of STC it has been shown how, with the use of proper trellis codes, multiple transmitting antennas can be exploited to improve system performance obtaining both diversity and coding gain, without sacrificing spectral efficiency [7], [8]. In particular, the design of STC over quasi-static flat fading (i.e., fading level constant over a frame and independent frame by frame) has been addressed in [8], where some handcrafted trellis codes for two transmitting antennas have been proposed. A number of extensions of this work have eventually appeared in the literature to design good codes for different scenarios. In [9], [10] a pragmatic approach to STC, called pragmatic space-time codes (P-STC), has been proposed: it simplifies the encoder and decoder structures and also allows a feasible method to search for good codes in block fading channels (BFC). P-STC consists in the use of common convolutional encoders and Viterbi decoders over multiple transmitting and receiving antennas, achieving maximum diversity and excellent performance, with no need of specific encoder or decoder different from those used for convolutional codes (CC); the Viterbi decoder requires only a simple modification in the metrics computation.

In this paper a design methodology of P-STCs for relay networks is provided, resulting in increased flexibility with respect to the above issues. For what concern the channel between transmitting and receiving antennas, we consider the BFC model [11] that represents a simple and powerful model to include a variety of fading rates, from ”fast” fading (i.e., ideal symbol interleaving) to quasi-static. Moreover, after the proposal of the P-STC structure for cooperative communication with various number of relays and transmitting antennas, we will derive pairwise error probability, asymptotic error probability bounds and design criteria to optimize diversity and coding gain. Finally, we will provide generators for good P-STC and show results with overlay construction over static channel and for various number of relays an various BFC.

II. System Model and Assumptions

The cooperation scheme is depicted in Fig. 1 and follows the time-division channel allocations with orthogonal cooperative diversity transmission [12]. Each user (i.e., the source) divides its own time-slot into two equal segments, the first from time t_1 to $t_1 + \Delta$ and the second from $t_2 = t_1 + \Delta$ to $t_2 + \Delta$, where Δ is the segment duration. In the first segment the source broadcasts its coded symbols; in the second all the active relays which are able to decode the source message forward the information through proper re-encoding trying to take advantage of the overall available diversity. Thus, the design of proper STCs for the two phases is crucial to maximize both achievable diversity and coding gain.

We assume n transmitting antennas at each terminal and m receiving antennas at the destination. So, $n_1 = n$ antennas will be used in the first phase and a total of $n_2 = Rn$ antennas will be used in the second phase, where R is the number of relays able to decode and forward the source message.

We indicate[1] with $c_{r,i}^{(t)}$ the modulation symbol transmitted by relay r ($0 \leq r \leq R$, and $r = 0$ is the source) on the antenna

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[1]The superscripts ^{H}, ^{T} and * denote conjugation and transposition, transposition only, and conjugation only, respectively.

i at discrete time t, i.e. at the $t^{\underline{\text{th}}}$ instant of the encoder clock. Each symbol is assumed to have unitary norm and generated according to the modulation format by proper mapping. Note that symbol $c_{0,i}^{(t)}$ is transmitted at time t_1+t, while symbols $c_{r,i}^{(t)}$ for $r>0$ are transmitted at time t_2+t. The received signals corresponding to all symbols $c_{r,i}^{(t)}$ are jointly processed by the decoder at the reference time t. We also denote with $\mathbf{C}^{(t)} = \left[c_{0,1}^{(t)}, c_{0,2}^{(t)}, \ldots, c_{R,n}^{(t)}\right]^T$ a super-symbol, which is the vector of the $(R+1)n$ outputs of the overall "virtual encoder" constituted by the source encoder and the relays encoders. A codeword is a sequence $\underline{c} = \left(\mathbf{C}^{(1)}, \ldots, \mathbf{C}^{(N)}\right)$ of N super-symbols generated by the source and relays encoders. This codeword $\underline{c}$ is interleaved before transmission to obtain the sequence $\underline{c}_I = \mathcal{I}(\underline{c}) = \left(\mathbf{C}^{(\sigma_1)}, \ldots, \mathbf{C}^{(\sigma_N)}\right)$, where $\sigma_1, \ldots, \sigma_N$ is a permutation of the integers $1, \ldots, N$ and $\mathcal{I}(\cdot)$ is the interleaving function. Note that with this notation the permutation is the same for all the transmitting terminals in the two phases.

The considered channel model includes additive white gaussian noise (AWGN) and multiplicative flat fading, with Rayleigh distributed amplitudes assumed constant over blocks of B consecutive transmitted space-time symbols and independent from block to block [11]. Perfect channel state information is assumed at the decoder for each node. The transmitted super-symbol at time σ_t goes through a compound channel described by the $(n_1+n_2)\times m$ channel matrix $\mathbf{H}^{(\sigma_t)} = [H_0^{(\sigma_t)}, \ldots, H_R^{(\sigma_t)}]^T$ where $H_r^{(\sigma_t)} = \left\{h_{r,i,s}^{(\sigma_t)}\right\}$, and $h_{r,i,s}^{(\sigma_t)}$ is the channel gain between transmitting antenna $i,\ i=1,\ldots,n$ of the terminal r and receiving antenna $s,\ s=1,\ldots,m$ at time σ_t. In the BFC model these channel matrices do not change for B consecutive transmissions, so that we actually have only $L = N/B$ possible distinct channel matrix instances per codeword[2]. By denoting with $\mathcal{Z} = \{\mathbf{Z}_1, \ldots, \mathbf{Z}_L\}$ the set of L channel instances, we have $\mathbf{H}^{(\sigma_t)} = \mathbf{Z}_l$ for $\sigma_t = (l-1)B+1, \ldots, lB$. When the fading block length, B, is equal to one, we have the ideally interleaved fading channel (i.e., independent fading levels from symbol to symbol), while for $L=1$ we have the quasi-static fading channel (fading level constant over a codeword); by varying L we can describe channels with different correlation degrees [11].

At the receiving side the sequence of received signal vectors is $\underline{r}_I = \left(\mathbf{R}^{(\sigma_1)}, \ldots, \mathbf{R}^{(\sigma_N)}\right)$, and after de-interleaving we have $\underline{r} = \mathcal{I}^{-1}(\underline{r}_I) = \left(\mathbf{R}^{(1)}, \ldots, \mathbf{R}^{(N)}\right)$, where the received vector at time t is $\mathbf{R}^{(t)} = \left[r_1^{(t,1)} r_1^{(t,2)} \cdots r_m^{(t,2)}\right]^T$ with components

$$r_s^{(t,1)} = \sqrt{E_s}\sum_{i=1}^{n} h_{0,i,s}^{(t)} c_{0,i}^{(t)} + \eta_s^{(t,1)}, \qquad s=1,\ldots,m\,, \quad (1)$$

in the first phase and

$$r_s^{(t,2)} = \sqrt{E_s}\sum_{r=1}^{R}\sum_{i=1}^{n} h_{r,i,s}^{(t)} c_{r,i}^{(t)} + \eta_s^{(t,2)}, \qquad s=1,\ldots,m\,, \quad (2)$$

[2]For the sake of simplicity we assume that N and B are such that L is an integer.

for the second phase. In this equations $r_s^{(t,l)}$ is the signal-space representation of the signal received by antenna s at time t in phase l, the noise terms $\eta_s^{(t,l)}$ are independent, identically distributed (i.i.d.) complex Gaussian random variables (r.v.s), with zero mean and variance $N_0/2$ per dimension, and the r.v.s $h_{r,i,s}^{(t)}$ represent the de-interleaved complex Gaussian fading coefficients. Since we assume spatially uncorrelated channels, these are i.i.d. with zero mean and variance $1/2$ per dimension, and, consequently, $|h_{r,i,s}^{(t)}|$ are Rayleigh distributed r.v.s with unitary power. The constellations are multiplied by a factor $\sqrt{E_s}$ in order to have a transmitted energy per symbol equal to E_s, which is also the average received symbol energy (per transmitting antenna) due to the normalization adopted on fading gains. Assuming the same symbol energy for every transmitter is motivated by the use of a power control technique which keeps constant received symbol energy averaged over fast fading. The energy transmitted per information bit is $E_b = E_s/(hR_c)$ where h is the number of bits per modulation symbol and R_c is the total code-rate of the cooperative space-time code.

III. SPACE-TIME CODES FOR COOPERATIVE RELAYING

In the case of the two-phase relaying scheme shown in Fig.1, the probability of failing the transmission over the two phases depends on the number of relays available for cooperation and on the link qualities between source and destination, between source and the R relays as well as between relays and destination. We assume that the set of relays is build up at the beginning of a data communication session and is kept unchanged over a long period of many slots. The set of relays is chosen by looking at active terminals able to guarantee a good average link quality (depending on terminal position and slow fading) with the source terminal. During this period a cooperative coding scheme is used by source and set of relays to protect the transmission of data frames between source and destination. The code components should be designed to maximize diversity and coding gain of the entire cooperative code for any possible number of cooperating relays [5].

In this paper we are considering the design of space-time trellis codes for relaying networks by using the pragmatic approach of [9], [10], which consists in a low-complexity architecture where the code components are build by the concatenation of a binary convolutional encoder and BPSK or QPSK modulator. This code architecture was also referred to as algebraic STC in [13]. Our "pragmatic" approach thus consists in using common convolutional codes as space-time codes, with the architecture presented in Fig. 2. Here, k information bits are encoded by a convolutional encoder with rate $k/(nh)$. The nh output bits are divided into n streams, one for each transmitting antenna, of binary phase shift keying (BPSK) ($h=1$) or quaternary phase shift keying (QPSK) ($h=2$) symbols that are obtained from a natural (Gray) mapping of h bits. If μ is the encoder constraint length then the associated trellis has $N_s = 2^{k(\mu-1)}$ states.

Differently from [10], the P-STC for cooperative communication are obtained by joining the $R+1$ code components used in cooperating transmitters, by using the trellis of each encoder (the same as for the CC), labelling the generic branch from state S_i to state S_j with the super-symbol $\widetilde{\mathbf{C}}_{S_i \to S_j} = [\widetilde{c}_{0,1}, \ldots, \widetilde{c}_{R,n}]^T$, where for BPSK $\widetilde{c}_{r,i}$ is the output of the $i^{\underline{\text{th}}}$ generator (in antipodal version) of the $r^{\underline{\text{th}}}$ transmitter.

One of the advantages of the pragmatic architecture is that the maximum likelihood (ML) decoder is the usual Viterbi decoder for the convolutional encoder adopted (same trellis), with a simple modification of the branch metrics as in Fig. 3. As additional advantage P-STC are easy to study and optimize, even over BFC. These advantages apply also when P-STC are used for cooperative communications, as will be further investigated in the next sections.

IV. Performance analysis in BFC

We first consider the derivation of the pairwise error probability (PEP). Given the transmitted codeword $\underline{c}$, the PEP, that is the probability that the ML decoder chooses the codeword $\underline{g} \neq \underline{c}$, conditional to the set of fading levels $\mathcal{Z}$, can be written as

$$\mathbb{P}\left\{\underline{c} \to \underline{g} | \mathcal{Z}\right\} = \frac{1}{2} \text{ erfc} \sqrt{\frac{E_s}{4N_0} d^2\left(\underline{c}, \underline{g} | \mathcal{Z}\right)}\,, \quad (3)$$

where

$$d^2\left(\underline{c}, \underline{g} | \mathcal{Z}\right) = \sum_{t=1}^{N} \sum_{s=1}^{m} \left[\left| \sum_{i=1}^{n} h_{0,i,s}^{(t)} \cdot \left(c_{0,i}^{(t)} - g_{0,i}^{(t)}\right) \right|^2 + \left| \sum_{r=1}^{R} \sum_{i=1}^{n} h_{r,i,s}^{(t)} \cdot \left(c_{r,i}^{(t)} - g_{r,i}^{(t)}\right) \right|^2 \right], \quad (4)$$

is the conditional Euclidean squared distance at the channel output [8]. In BFC we first rewrite the squared distance as follows

$$d^2\left(\underline{c}, \underline{g} | \mathcal{Z}\right) = \sum_{t=1}^{N} \sum_{s=1}^{m} \mathbf{h}_s^{(t)} \mathbf{A}^{(t)}(\underline{c}, \underline{g}) \mathbf{h}_s^{(t)H}\,, \quad (5)$$

where $\mathbf{h}_s^{(t)} = \left[h_{0,1,s}^{(t)}, h_{0,2,s}^{(t)}, ..., h_{R,n,s}^{(t)}\right]$ is the $1 \times (R+1)n$ vector of the fading coefficients related to the receiving antenna s. In (5) the $(n_1+n_2) \times (n_1+n_2)$ matrix $\mathbf{A}^{(t)}(\underline{c}, \underline{g})$ is Hermitian non-negative definite [10] with block structure

$$\mathbf{A}^{(t)}(\underline{c}, \underline{g}) = \begin{bmatrix} \mathbf{a}^{(t,1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{a}^{(t,2)} \end{bmatrix}$$

where $\mathbf{a}^{(t,1)} = \left(\mathbf{c}^{(t,1)} - \mathbf{g}^{(t,1)}\right)\left(\mathbf{c}^{(t,1)} - \mathbf{g}^{(t,1)}\right)^H$ and $\mathbf{a}^{(t,2)} = \left(\mathbf{c}^{(t,2)} - \mathbf{g}^{(t,2)}\right)\left(\mathbf{c}^{(t,2)} - \mathbf{g}^{(t,2)}\right)^H$, after having split the generic super-symbol $\mathbf{C}^{(t)}$ in the two parts transmitted during phase 1 and phase 2 respectively, i.e. $\mathbf{C}^{(t)} = [\mathbf{c}^{(t,1)}\mathbf{c}^{(t,2)}]^T$ where $\mathbf{c}^{(t,1)} = [c_{0,1}, ..., c_{0,n}]$ and $\mathbf{c}^{(t,2)} = [c_{1,1}, ..., c_{R,n}]$. Due to the BFC assumption, for each frame and each receiving antenna the fading channel is described by only L different vectors $\mathbf{h}_s^{(t)} \in \left\{\mathbf{z}_s^{(1)}, \mathbf{z}_s^{(2)}, \ldots, \mathbf{z}_s^{(L)}\right\}$, $s = 1, \ldots, m$, where $\mathbf{z}_s^{(l)}$ is the s-th row of $\mathbf{Z}_l$. By grouping vectors, we can rewrite (5) as

$$d^2\left(\underline{c}, \underline{g} | \mathcal{Z}\right) = \sum_{l=1}^{L} \sum_{s=1}^{m} \mathbf{z}_s^{(l)} \mathbf{F}^{(l)}(\underline{c}, \underline{g}) \mathbf{z}_s^{(l)\,H}\,, \quad (6)$$

where

$$\mathbf{F}^{(l)}(\underline{c}, \underline{g}) \triangleq \sum_{t \in T(l)} \mathbf{A}^{(t)}(\underline{c}, \underline{g}) \;\; l = 1, \ldots, L \quad (7)$$

and $T(l) \triangleq \{t : \mathbf{H}^{(\sigma_t)} = \mathbf{Z}_l\}$ is the set of indexes t where the channel fading gain matrix is equal to $\mathbf{Z}_l$. This set depends on the interleaving strategy adopted. Note that in our scheme (Fig. 2) the interleaving is done "horizontally" for each transmitting antenna and in the same way for each transmitter, and that the set $T(l)$ is independent on s (i.e., the interleaving rule is the same for all antennas).

The matrix $\mathbf{F}^{(l)}(\underline{c}, \underline{g})$ is also Hermitian non-negative definite, being the sum of Hermitian non-negative definite matrices. It has, therefore, real non-negative eigenvalues. Moreover, it can be written as $\mathbf{F}^{(l)}(\underline{c}, \underline{g}) = \mathbf{U}^{(l)}\mathbf{\Lambda}^{(l)}\mathbf{U}^{(l)H}$, where $\mathbf{U}^{(l)}$ is a unitary matrix and $\mathbf{\Lambda}^{(l)}$ is a real diagonal matrix, whose diagonal elements $\lambda_i^{(l)}$ with $i = 1, \ldots, \tilde{n} = n_1 + n_2$ are the eigenvalues of $\mathbf{F}^{(l)}(\underline{c}, \underline{g})$ counting multiplicity. Note that $\mathbf{F}^{(l)}$ and its eigenvalues $\lambda_i^{(l)}$ are a function of $\underline{c} - \underline{g}$. As a result, we can express the squared distance $d^2\left(\underline{c}, \underline{g} | \mathcal{Z}\right)$ by utilizing the eigenvalues of $\mathbf{F}^{(l)}(\underline{c}, \underline{g})$ as follows:

$$d^2\left(\underline{c}, \underline{g} | \mathcal{Z}\right) = \sum_{l=1}^{L} \sum_{s=1}^{m} \mathbf{B}_s^{(l)} \mathbf{\Lambda}^{(l)} \mathbf{B}_s^{(l)\,H} = \sum_{l=1}^{L} \sum_{s=1}^{m} \sum_{i=1}^{\tilde{n}} \lambda_i^{(l)} \left|\beta_{i,s}^{(l)}\right|^2$$

where $\mathbf{B}_s^{(l)} = \left[\beta_{1,s}^{(l)}, \beta_{2,s}^{(l)}, ..., \beta_{\tilde{n},s}^{(l)}\right] = \mathbf{z}_s^{(l)}\mathbf{U}^{(l)}$. It should be observed that the form of matrix $\mathbf{F}^{(l)}(\underline{c}, \underline{g})$ is different from the matrix of the same space-time code working on a system with $n_1 + n_2$ transmit antennas defined in [10] due to the use of two distinct transmission phases in the cooperative system. Therefore, the same code used in the cooperative system may achieve different diversity and coding gains. It should also be noted that this matrix is diagonal (hence full-rank) only when $n = 1$ and $R = 1$: when transmitters have more than one antenna, or more than one relay cooperate to transmission, only a suitable choice of the code may lead to a full rank matrix, as shown later.

The unconditional pairwise error probability (PEP) becomes $\mathbb{P}\left\{\underline{c} \to \underline{g}\right\} = \mathbb{E}\left\{\mathbb{P}\left\{\underline{c} \to \underline{g} | \mathcal{Z}\right\}\right\}$ where $\mathbb{E}\{.\}$ indicates expectation with respect to fading. By evaluating the asymptotic behavior of it for large signal-to-noise ratio (SNR) we obtain (see [14])

$$\mathbb{P}\left\{\underline{c} \to \underline{g}\right\} \leq K(m\eta) \left[\prod_{l=1}^{L} \prod_{i=1}^{\eta_l} \lambda_i^{(l)} \left(\frac{E_s}{4N_0}\right)^{\eta}\right]^{-m} \quad (8)$$

where[3] $K(d) = \frac{1}{2^{2d}}\binom{2d-1}{d}$, the integer $\eta_l = \eta_l(\underline{c}, \underline{g})$ is the number of non-zero eigenvalues of $\mathbf{F}^{(l)}(\underline{c}, \underline{g})$, and η (that we can call the pairwise transmit diversity) is the sum of the ranks of $\mathbf{F}^{(l)}(\underline{c}, \underline{g})$, i.e.

$$\eta = \eta(\underline{c}, \underline{g}) = \sum_{l=1}^{L} \text{rank}\left[\mathbf{F}^{(l)}(\underline{c}, \underline{g})\right] = \sum_{l=1}^{L} \eta_l\,. \quad (9)$$

The PEP between $\underline{c}$ and $\underline{g}$ shows a diversity $m\eta$ that is the product of transmit and receive diversity. By using the

[3] A looser bound can be obtained by observing that $K(d) \leq 1/4$.

asymptotic approximation (8), and by observing that the error probability is upper bounded by $\sum_{\underline{c}}\sum_{\underline{g}\neq\underline{c}}\mathbb{P}\left\{\underline{c}\right\}\mathbb{P}\left\{\underline{c}\rightarrow\underline{g}\right\}$ and the retained dominant terms are those with transmit diversity $\tilde{\eta}_{\min}=\min_{\underline{c}}\eta_{\min}(\underline{c})$, where $\eta_{\min}(\underline{c})=\min_{\underline{g}}\eta(\underline{c},\underline{g})$, the asymptotic error probability bound can be written

$$\tilde{P}_{w_\infty}\approx K(\tilde{\eta}_{\min}m)\left(\frac{E_s}{4N_0}\right)^{-\tilde{\eta}_{\min}\cdot m}\tilde{F}_{\min}(m)\,. \qquad (10)$$

From (10) we observe that the asymptotic performance of STC over BFC depends on both the achievable diversity, $\tilde{\eta}_{\min}\cdot m$, and the performance factor (related to the coding gain):

$$\tilde{F}_{\min}(m)\triangleq\sum_{\underline{c}}\mathbb{P}\left\{\underline{c}\right\}\sum_{\underline{g}\in\mathcal{E}(\underline{c},\tilde{\eta}_{\min})}\left[\prod_{l=1}^{L}\prod_{i=1}^{\eta_l}\lambda_i^{(l)}\right]^{-m}. \qquad (11)$$

where $\mathcal{E}(\underline{c},x)=\left\{\underline{g}\neq\underline{c}:\eta(\underline{c},\underline{g})=x\right\}$ is the set of codeword sequences at minimum diversity.

To summarize, the derivation of the asymptotic behavior of a given STC with a given length requires computing the matrices $\mathbf{F}^{(l)}(\underline{c},\underline{g})$ in (7) with their rank and product of non-zero eigenvalues. Moreover, according to [15], by restricting in the bound the set of sequences $\underline{g}$ to those corresponding to paths in the trellis diagram of the code diverging only once from the path of codeword $\underline{c}$, the union bound becomes tighter and can be evaluated in an effective way, by using the methodology illustrated in [10] through the concept of space-time generalized transfer function.

V. PRAGMATIC SPACE-TIME CODES DESIGN FOR RELAYING

We now address design criteria for good cooperative STC and efficient search for the optimum (in the sense defined later) generators for the code components of cooperative STC in BFC. For the design of a good cooperative STC, we consider the two following situations:

1) By assuming that the cooperative code is working with a predefined number of cooperating relays R, it may be designed as P-STC with k binary inputs and $n(R+1)$ output sysmbols which maximize diversity gain and coding gain.[4] We refer to this solution as cooperative pragmatic space-time codes (CP-STC). This design method does not guarantee that the first rate k/nh component of the code used in phase 1 is the best performing code, as well as it does not guarantee good performance when some code components are not used by relays not able in some frames to decode the source message.

2) By assuming that the cooperative code is obtained by joining code components in phase 2 from every relay able to decode the source message, the code may be designed as STC with overlay construction [16]. We refer to this construction as cooperative overlay pragmatic space-time codes (COP-STC). With this method, a good code for R relays is designed starting from a good code for $R-1$ relays and by adding the best code component that maximize diversity and coding gain of the final code. In this way the first code component used by the source in phase 1 is always a good code. For fixed set of cooperating relays the sequence of additional code components can be orderly assigned to the relays ranked according the average link quality, in such a way that the second code component is assigned to the relay with the best link quality and so on, so that they are used with high probability in the same combinations for which they have been designed. Moreover, is more easy to design the additional code components than the entire cooperative code.

The design of STC with overlay construction was addressed in [16], but not in the special case of cooperative codes. They derived algebraic design criteria aimed at maximizing diversity gain, without addressing coding gain issues. The work in [5] proposed to use this STC with overlay construction as a cooperative STC but without specializing the design for the cooperative scenario. We propose a STC code search criterion suitable for both the outlined design possibility, i.e. the design of an entire rate $k/(nh(R+1))$ P-STC and the design of rate k/nh code components in an overlay structure.

Our search criterion is based on the asymptotic error probability in (10), so that, among the set of non-catastrophic codes, the optimum code with fixed parameters (n,k,h,μ):

- maximizes the achieved diversity, $\tilde{\eta}_{\min}$;
- minimizes the performance factor $\tilde{F}_{\min}(m)$;

where the values of $\tilde{\eta}_{\min}$ and $\tilde{F}_{\min}(m)$ can be extracted from the space-time generalized transfer function (ST-GTF) of the code. Therefore, an exhaustive search algorithm should evaluate the ST-GTF for each code of the set.

Usually, in the literature a method based on the evaluation of the worst PEP is considered. Although the worst PEP carries information about the achievable diversity, $\tilde{\eta}_{\min}$, it is incomplete with respect to coding gain, thus producing a lower bound for the error probability. Even though our method based on the union bound is still approximate with respect to coding gain (giving an upper bound) it includes more information than the other method. This approach gives good results in reproducing the correct performance ranking of the codes among those achieving the same diversity $\tilde{\eta}_{\min}$, as will be checked in the numerical results section. Of course, the achievable diversity is the most important design parameter. Since $\tilde{\eta}_{\min}$ can not be larger than both $\eta(\underline{c},\underline{g})\leq(R+1)nL$ and the free distance d_f of the convolutional code used to build the P-STC, it appears that to capture the maximum diversity per receiving antenna offered by the channel, $(R+1)nL$, the free distance of a good code for a given BFC should be at least $(R+1)nL$ or larger. On the other hand, there is a fundamental limit on the achievable diversity related to the Singleton bound for BFC [11].

VI. RESULTS

In this section we first report the results obtained for the search of good cooperative STC with overlay construction for different system configurations obtained with $R=1,2,3$ relays, $n=1,2$ transmitting antennas, $m=1$ receiving antennas. All the codes proposed are full diversity codes.

[4] A suboptimal pragmatic solution to this problem may be to build the code with the rate $k/(nh(R+1))$ maximum distance convolutional code optimum for AWGN channel.This solution may be not optimal even in terms of diversity gain and therefore should be checked by means of simulations. However, we observed that in many case it leads to quite good results when R relays are active in the second phase.

Two approaches are considered for overlay construction: the first considers the use of best (for AWGN channel) maximum distance rate k/hn code as a first code component; the second considers as the first code component the best rate k/hn P-STC also reported in [10]. When possible these codes are compared with the cooperative STC proposed in [5].

The results for QPSK modulation are collected in Tables I-III. As a relevant comment we can note that the codes proposed are able to capture the maximum available diversity already with few states in the trellis. By increasing the number of trellis states only a small coding gain improvement is obtained. It is worth noting that cooperative codes obtained by using the best P-STC as a first code component usually perform better than the others, including the best available from the literature [5]. It is also found in Tab. III that the 4 state code in [5] for $R=2$ does not achieve full diversity.

Suboptimal cooperative STC, working with a predefined number of cooperating relays R and constructed by pragmatically choosing the best maximum distance convolutional codes for AWGN. lead in most of the cases to good or acceptable results. However, they sometimes do not achieve full diversity. As an example this is the case of the rate 1/4, 4-state code with generators $(5,7,7,7)_8$ for systems with BPSK, $n=2$ and $R=1$ cooperating relays, which achieves a maximum diversity of 3. According to Singleton bound, note that full diversity rate $k/(h(R+1)n)$ codes can be constructed if $k \leq h$.

We now report the performance results, in terms of frame error rate (FER) as a function of SNR for CP-STC and COP-STC in different conditions and error-free source-ralay link. The SNR is defined as E_b/N_0 per receiving antenna element where, for a fair comparison among situations with different number of relays, E_b is the total energy per information bit over all transmitting nodes and averaged with respect to fading. We verify the performance of COP-STC codes for QPSK modulation obtained through the design and search criterion explained in Sec. V. As an example, we propose here the generators for rate $1/(4R)$, 8 state codes obtained through two different approaches for overlay construction, consisting in designing the overall code starting from first code component taken as the the best rate $1/4$ code for AWGN, in one case, or as the best P-STC. The former has generators $(13,15,15,17)_8$ for source and $(11,17,16,12)_8$ for relay, while the latter has generators $(11,15,17,13)_8$ and $(06,15,13,12)_8$, respectively. In Fig. 4 we show the performance without relaying and with one relay in quasi-static fading channel (i.e., $L=1$). Note that the best results are obtained with the second approach with a performance gain in agreement with the values of the performance factor ($\tilde{F}_{min}(1)/N$ is 0.00069 and 0.00053, respectively). Finally, we investigate in Fig. 5 the impact of the number of relaying nodes, ranging from 0 to 3, for the 8 state COP-STC with QPSK with one transmitting antenna per node and one receiving antenna in quasi static fading channel. The generators, obtained with the search criterion proposed here, for source and 3 relays are respectively given by $(15,17)_8$, $(11,13)_8$, $(05,16)_8$, $(16,13)_8$. The performance factor $\tilde{F}_{min}(1)$ shows a 35% reduction of error probability with respect to the codes proposed in [5].

VII. Conclusions

In this paper we investigated the feasibility of a pragmatic approach to space-time codes for wireless cooperative relay networks, where common convolutional encoders and decoders are used with suitably defined branch metrics. We also proposed a design criterion to rank different codes based on the asymptotic error probability union bound. A search methodology to obtain optimum generators for different fading rates has then been given in BFC. It is shown that P-STCs applied to cooperative communication systems achieve good performance and that they are suitable for systems with different spectral efficiencies, number of antennas and fading rates, making them a valuable choice both in terms of implementation complexity and performance.

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity. part i. system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[2] J. Laneman and G. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *Information Theory, IEEE Transactions on*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[3] Z. Lin, E. Erkip, and A. Stefanov, "Cooperative regions and partner choice in coded cooperative systems," *Communications, IEEE Transactions on*, vol. 54, no. 7, pp. 1323–1334, July 2006.

[4] J. Luo, R. Blum, L. Cimini, L. Greenstein, and A. Haimovich, "Decode-and-forward cooperative diversity with power allocation in wireless networks," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 3, pp. 793–799, March 2007.

[5] A. Stefanov and E. Erkip, "Cooperative space-time coding for wireless networks," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1804– 1809, Nov. 2005.

[6] M. Dohler, Y. Li, B. Vucetic, A. H. Aghvami, M. Arndt, and D. Barthel, "Performance analysis of distributed space-time block-encoded sensor networks," *Vehicular Technology, IEEE Transactions on*, vol. 55, no. 6, pp. 1776–1789, Nov. 2006.

[7] J.-C.Guey, M.P.Fitz, M.R.Bell, and W.-Y. Kuo, "Signal design for transmitter diversity wireless communication systems over rayleigh fading channels," *Proc. 46th Annual Int. Veh. Technol. Conf.*, pp. 136–140, Sep. 1996, atlanta, GA.

[8] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.

[9] M. Chiani, A. Conti, and V. Tralli, "A pragmatic approach to space-time coding," in *Proc. IEEE Int. Conf. on Commun.*, vol. 9, Helsinki, FI, Jun. 2001, pp. 2794 – 2799.

[10] ——, "Pragmatic space-time trellis codes for block fading channels," *IEEE Trans. Inform. Theory*, 2007, submitted, also available at http://arxiv.org/abs/cs/0703142v1.

[11] E. Malkamaki and H.Leib, "Coded diversity on block-fading channels," *IEEE Trans. Inform. Theory*, vol. 45, no. 2, pp. 771–781, Mar. 1999.

[12] A. Stefanov and E. Erkip, "Cooperative coding for wireless networks," *Communications, IEEE Transactions on*, vol. 52, no. 9, pp. 1470–1476, Sept. 2004.

[13] H. El Gamal and A.R.Hammons, "On the design of algebraic space-time codes for MIMO block-fading channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 1, pp. 151–163, Jan. 2003.

[14] M. Chiani, A. Conti, and V. Tralli, "Further results on convolutional code search for block-fading channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 6, pp. 1312–1318, Jun. 2004.

[15] G.Caire and E.Viterbo, "Upper bound on the frame error probability of terminated trellis codes," *IEEE Commun. Lett.*, vol. 2, no. 1, pp. 2–4, Jan. 1998.

[16] H. E. Gamal, A. R. Hammons, and A. Stefanov, "Space-time overlays for convolutionally coded systems," *IEEE Trans. Commun.*, vol. 51, no. 9, pp. 1603–1612, Sept. 2003.

TABLE III
OPTIMUM OVERLAYS FOR RATE $1/(2R)$ COP-STC WITH QPSK, $n=1$, $m=1$, $R \in \{2,3,4\}$ ON QUASI-STATIC CHANNEL. THE BASIC CODE FOR SINGLE TRANSMITTER IS THE BEST CONVOLUTIONAL CODE FOR AWGN CHANNEL. THE BOTTOM PART OF THE TABLE REFERS TO C-STC AS IN [5]

μ	**Gen.** $r=0$	**Gen.** $r=1$	**Gen.** $r=2$	**Gen.** $r=3$	$\tilde{F}_{\min}(1)/N$ $R=1$	$\tilde{F}_{\min}(1)/N$ $R=2$	$R=3$	FER ($R=1$ @12dB)	FER ($R=2$ @12dB)
2	$(1,3)_8$	$(1,3)_8$	$(2,1)_8$	$(2,2)_8$	0.060	0.012	div <4	0.041	0.040
3	$(5,7)_8$	$(1,3)_8$	$(6,4)_8$	$(2,1)_8$	0.101	0.0113	0.00237	0.026	0.012
4	$(15,17)_8$	$(11,13)_8$	$(05,16)_8$	$(16,13)_8$	0.189	0.0136	0.00148	0.015	0.0051
5	$(23,35)_8$	$(27,31)_8$	$(21,37)_8$		0.315	0.0157		0.013	0.0038
3	$(5,7)_8$	$(5,7)_8$	$(5,7)_8$	-	0.125	div <3	-	0.020	0.022
4	$(15,17)_8$	$(13,15)_8$	$(17,13)_8$	-	0.323	0.0191	-	0.015	0.0069
5	$(23,35)_8$	$(25,37)_8$	$(27,33)_8$	-	0.401	0.0169	-	0.013	0.038

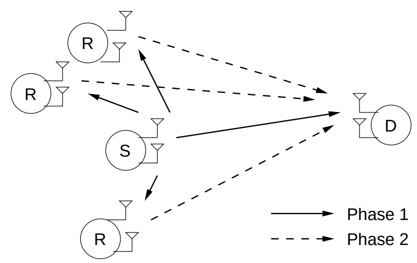

Fig. 1. Two phases relaying network: phase 1 (continuous line), phase 2 (dashed line). Source, relays and destination nodes are denoted with S, R, D, respectively.

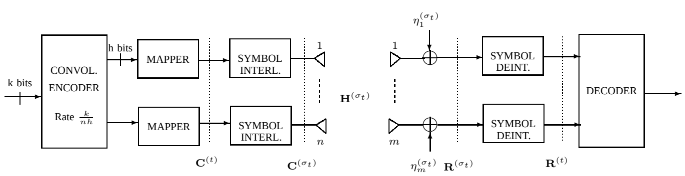

Fig. 2. Architecture of pragmatic space-time codes. In cooperative P-STC, instead of n we must consider the overall number of antennas $(R+1)n$ and the convolutional encoder is the ensamble of single encoders, one for each transmitter (source + R relays).

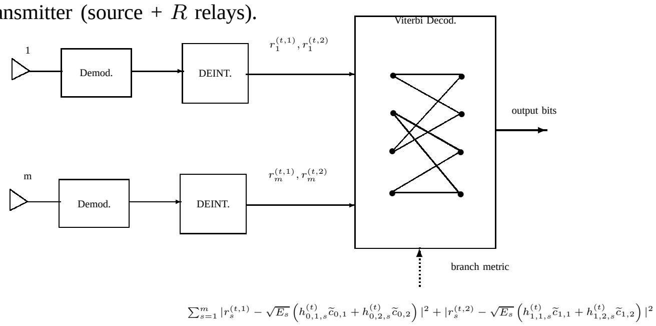

Fig. 3. Receiver structure for the proposed P-STCs. The Viterbi decoder is the classical with the only change that the metric on a generic branch is, for $n=2$ and $R=1$, $\sum_{s=1}^{m} |r_s^{(t,1)} - \sqrt{E_s}\left(h_{0,1,s}^{(t)}\tilde{c}_{0,1} + h_{0,2,s}^{(t)}\tilde{c}_{0,2}\right)|^2 + |r_s^{(t,2)} - \sqrt{E_s}\left(h_{1,1,s}^{(t)}\tilde{c}_{1,1} + h_{1,2,s}^{(t)}\tilde{c}_{1,2}\right)|^2$, being $\tilde{c}_{0,1}, \tilde{c}_{0,2}, \tilde{c}_{1,1}, \tilde{c}_{1,2}$, the four symbols associated to the branch: $r_s^{(t,l)}$ is received at time $t_l + t$, $l=1,2$.

TABLE I
OPTIMUM OVERLAYS FOR RATE $1/(2R)$ COP-STC WITH QPSK, $n=2$, $m=1$, $R=2$ ON QUASI-STATIC CHANNEL. THE BASIC CODE FOR SINGLE TRANSMITTER IS A STC AS IN [10].

μ	**Generators** $r=0$	**Generators** $r=1$	$\tilde{F}_{\min}(1)/N$
2	$(1,2,3,1)_8$	$(1,2,3,1)_8$	0.00305
3	$(2,5,7,6)_8$	$(2,7,5,3)_8$	0.00109
4	$(11,15,17,13)_8$	$(06,15,13,12)_8$	0.00053

TABLE II
OPTIMUM OVERLAYS FOR RATE $1/(2R)$ WITH QPSK, $n=2$, $m=1$, $R=2$ ON QUASI-STATIC CHANNEL. THE BASIC CODE FOR SINGLE TRANSMITTER IS A STC WITH BEST CONVOLUTIONAL CODE FOR AWGN CHANNEL.

μ	**Generators** $r=0$	**Generators** $r=1$	$\tilde{F}_{\min}(1)/N$
2	$(1,3,3,3)_8$	$(1,2,3,1)_8$	0.0075
3	$(5,7,7,7)_8$	$(2,6,5,3)_8$	0.00239
4	$(13,15,15,17)_8$	$(11,17,16,12)_8$	0.00069

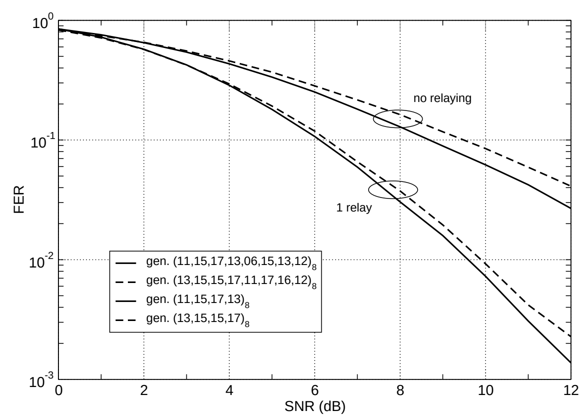

Fig. 4. QPSK, 8 states, generators as in Tables I and II without and with one relay, 2 transmitting antennas per node, 1 receiving antenna, in quasi-static fading channel.

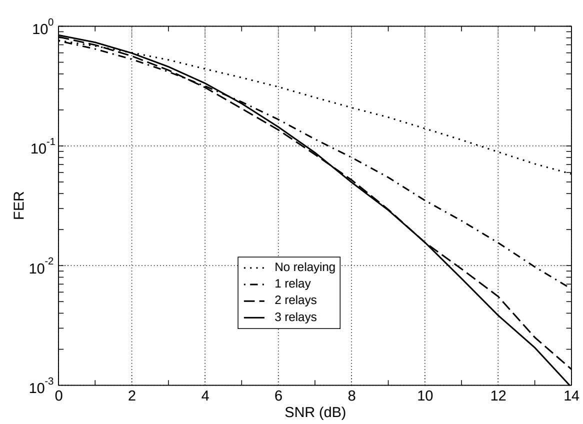

Fig. 5. QPSK, 8 states, generators as in Tab. III, various number of relaying nodes, 1 transmitting antenna per node, 1 receiving antenna, in quasi-static fading channel. All the codes achieve asymptotic slope of one decade every $10/(R+1)$ dBs.